# Dynamical mass in the solar neighborhood

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# "Weighting" the Galactic disk

# by means of the kinematics of its stellar components

Ancient art, dating back to Kapteyn (1922, ApJ, 55, 302)

and Oort (1932, BAN, 6, 249)

Aimed to detect invisible matter

Models of spherical dark matter halo:

$\rho_{DM,o} > 5\ mM_\odot\ pc^{-3}$

$\rho_{DM,o} = 8\text{-}10\ mM_\odot\ pc^{-3}$

Assumed in direct detection experiments

# *Poisson Equation:*

$$\nabla^2 \Phi = 4\pi G \rho_{\mathrm{tot}}$$

$$-4\pi G \Sigma(Z) = \int_{-Z}^{Z} \frac{1}{R} \frac{\partial}{\partial R}(R F_R) dz + 2 \cdot [F_z(Z) - F_z(0)]$$

*relates mass to potential or force*

# *Jeans Equation:*

$$F_R = -\frac{\partial \phi}{\partial R} = \frac{1}{\rho}\frac{\partial(\rho\overline{U^2})}{\partial R} + \frac{1}{\rho}\frac{\partial(\rho\overline{UW})}{\partial Z} + \frac{\overline{U^2} - \overline{V^2}}{R} + \frac{1}{\rho}\frac{\partial(\rho\overline{U})}{\partial t},$$

$$F_Z = -\frac{\partial \phi}{\partial Z} = \frac{1}{\rho}\left[\frac{\partial(\rho\overline{W^2})}{\partial Z} + \frac{\rho\overline{UW}}{R} + \frac{\partial(\rho\overline{UW})}{\partial R} + \frac{\partial(\rho\overline{W})}{\partial t}\right]$$

*relates potential or force*

*to kinematics and spatial distribution*

# Kuijken & Gilmore (1989, MNRAS, 239, 57; KG89):

*distribution function of energy:*

$$f_z(z, v_z) = f_z(E_z) \qquad E_z = \psi(z) + \tfrac{1}{2}v_z^2$$

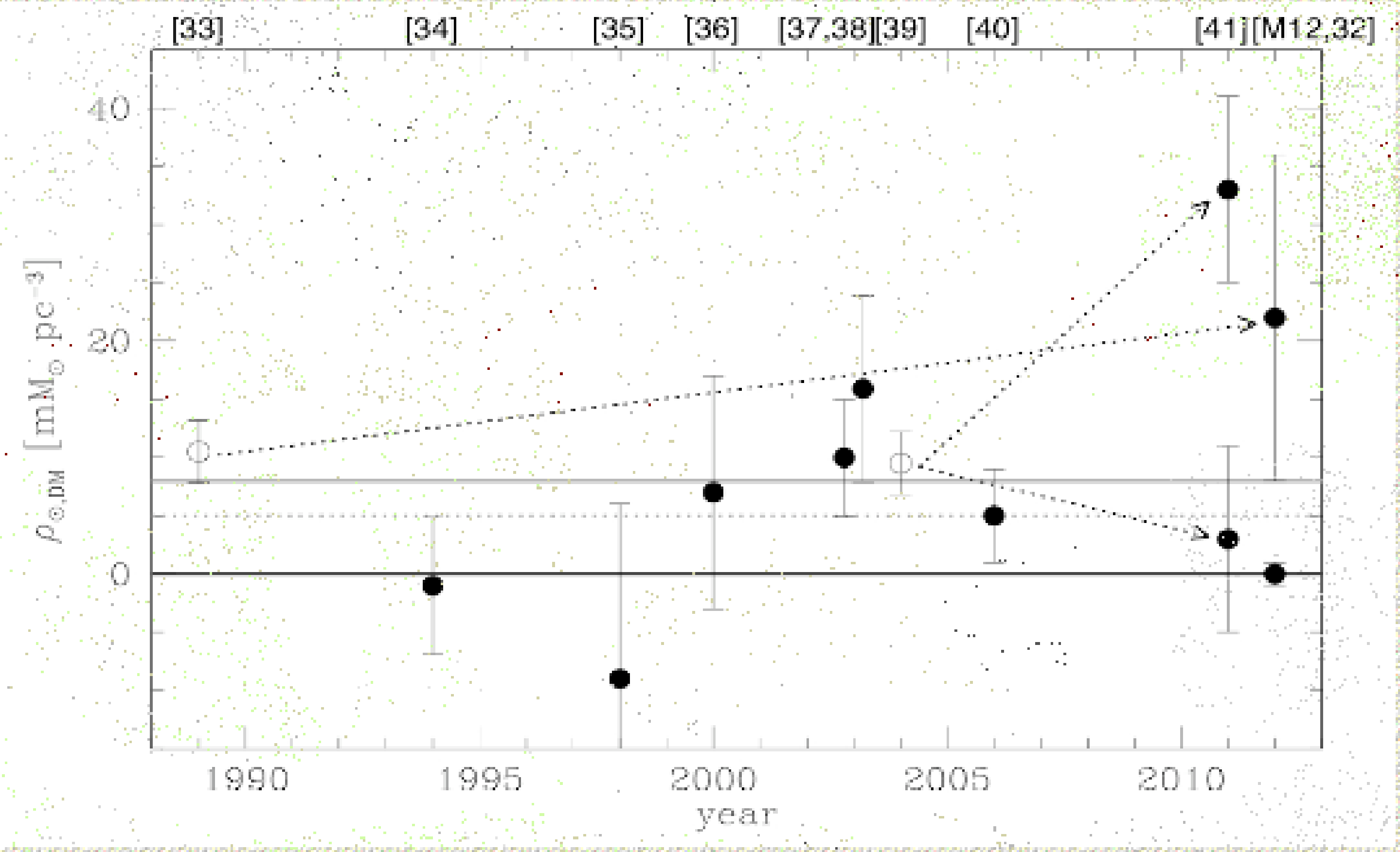
$$f_z(E_z) = \frac{1}{\pi}\int_{E_z}^{\infty} \frac{-d\nu/d\psi}{\sqrt{2(\psi - E_z)}}\, d\psi$$

- Model the potential
- Deduce f(E) from density distribution
- Deduce best-fit potential parameters comparing with observations

Extensive literature with variations of the KG89 method

- Kuijken & Gilmore (1989, MNRAS, 239, 57)
- Flynn & Fuchs (1994, MNRAS, 270, 471)
- Crezé et al. (1998, A&A, 329, 920)
- Holmberg & Flynn (2000, MNRAS, 313, 209)
- Siebert et al. (2003, A&A, 399, 531)
- Holmberg & Flynn (2004, MNRAS, 352, 440)
- Bienaymé et al. (2006, A&A, 446, 933)
- Garbari et al. (2011, MNRAS, 416, 2318)
- Garbari et al. (2012, MNRAS, 425, 1445)
- Zhang et al. (2012, arXiv:1209.0256)

# General agreement?



## Drawbacks :

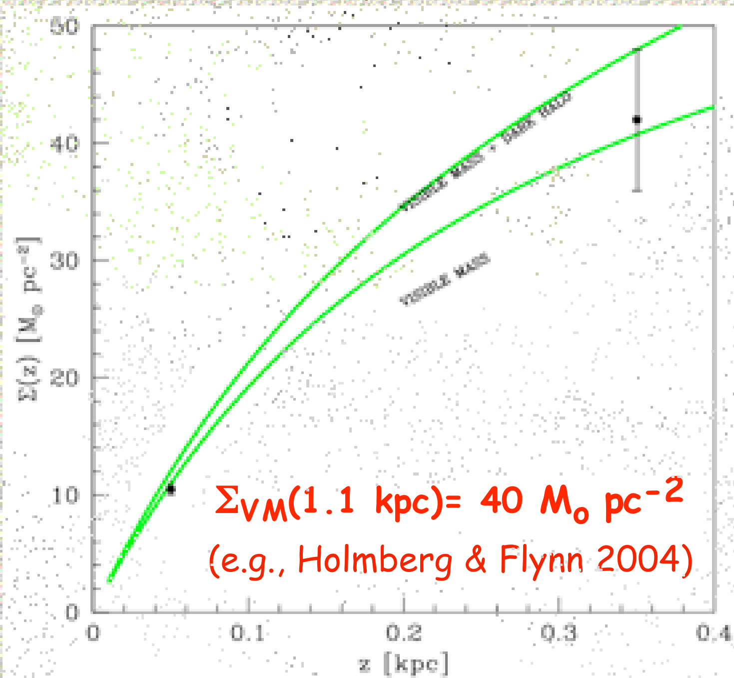
- 1D or pseudo-1D approximation:
valid only close to the plane, visible matter dominates

- No direct measurement, but modelling

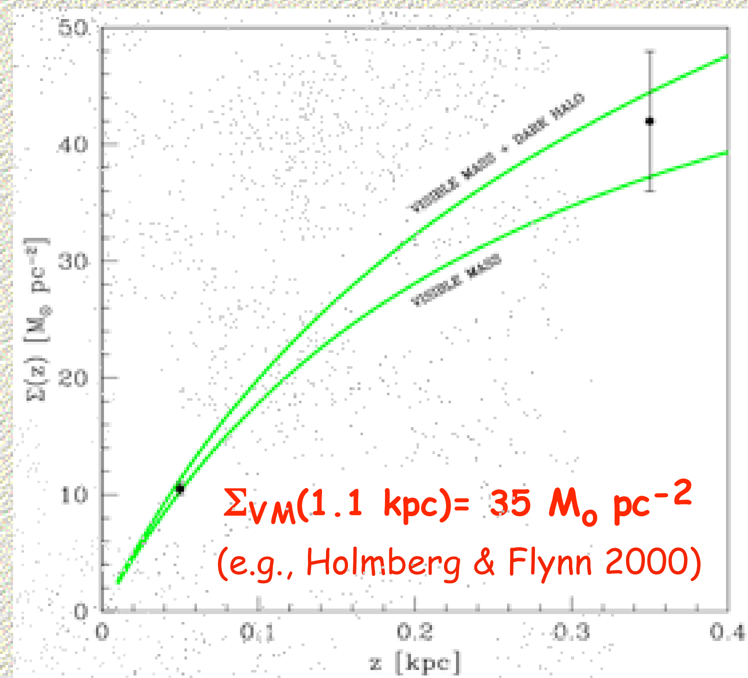
- Dark matter halo is a prior in the calculation

# Exception: Korchagin et al. (2003, AJ, 126, 2896)

## Direct measurement, analytical expression, many approximations




$$\Sigma_{\rm out}(z_{\rm out}) = 2\int_0^{z_{\rm out}} \rho(R,z)\,dz$$
$$= -\frac{F_z(R, z_{\rm out})}{2\pi G} + \frac{2z_{\rm out}(B^2 - A^2)}{2\pi G}$$



Our aim:

Expansion of Korchagin et al.'s formulation,

NO near-plane approximations

- Analytical expression (no best-fit model)
- No prior assumption on the mass distribution
- Valid at any Galactic height (and Galactocentric distance)

# Basic hypotheses:

, 747, 101)

1. Steady state
2. Vertical force null on the plane
3. No net radial and vertical motion
4. Symmetry of dispersions w.r.t. the plane
5. Antisymmetry of UW w.r.t. the plane
6. Double-exponential disk density distribution ($h_z$, $h_R$)
7. $h_R$ constant with z
8. No disk flare

$$-2\pi G\Sigma(R,z) = \frac{\partial\sigma_W^2}{\partial z} - \frac{\sigma_W^2}{h_{z,\rho}} - \int_0^z \frac{\sigma_U^2}{R h_{R,\rho}} dz + \int_0^z \frac{\partial^2\sigma_U^2}{\partial R^2} dz + \int_0^z \left(\frac{2}{R} - \frac{1}{h_{R,\rho}}\right) \cdot \frac{\partial\sigma_U^2}{\partial R} dz +$$
$$+ \frac{2}{R}\int_0^Z \overline{V}(\partial_R \overline{V}) dz - \frac{1}{R}\int_0^z \frac{\partial\sigma_V^2}{\partial R} dz + \overline{UW}\left(\frac{2}{R} - \frac{1}{h_{R,\rho}}\right) + 2\frac{\partial\overline{UW}}{\partial R} - \frac{1}{h_{z\rho}}\int_0^z \frac{\partial\overline{UW}}{\partial R} dz.$$

# Specific case: Moni Bidin et al. (2012, ApJ, 747, 101)

no radial information

9. Rotational velocity constant with R at all z
10. The square of the dispersions radially decay exponentially following the mass distribution

$$\Sigma(z) = \frac{1}{2\pi G}\left[k_1 \cdot \int_0^z \sigma_U^2 dz + k_2 \cdot \int_0^z \sigma_V^2 dz + k_3 \cdot \overline{UW} + \frac{\sigma_W^2}{h_{z,\rho}} - \frac{\partial \sigma_W^2}{\partial z}\right],$$

$$k_1 = \frac{3}{R_\odot \cdot h_{R,\rho}} - \frac{2}{h_{R,\rho}^2},$$

$$k_2 = -\frac{1}{R_\odot \cdot h_{R,\rho}},$$

$$k_3 = \frac{3}{h_{R,\rho}} - \frac{2}{R_\odot}.$$

If kinematics is measured only at $z>x$

Integration requires extrapolation to lower $z$

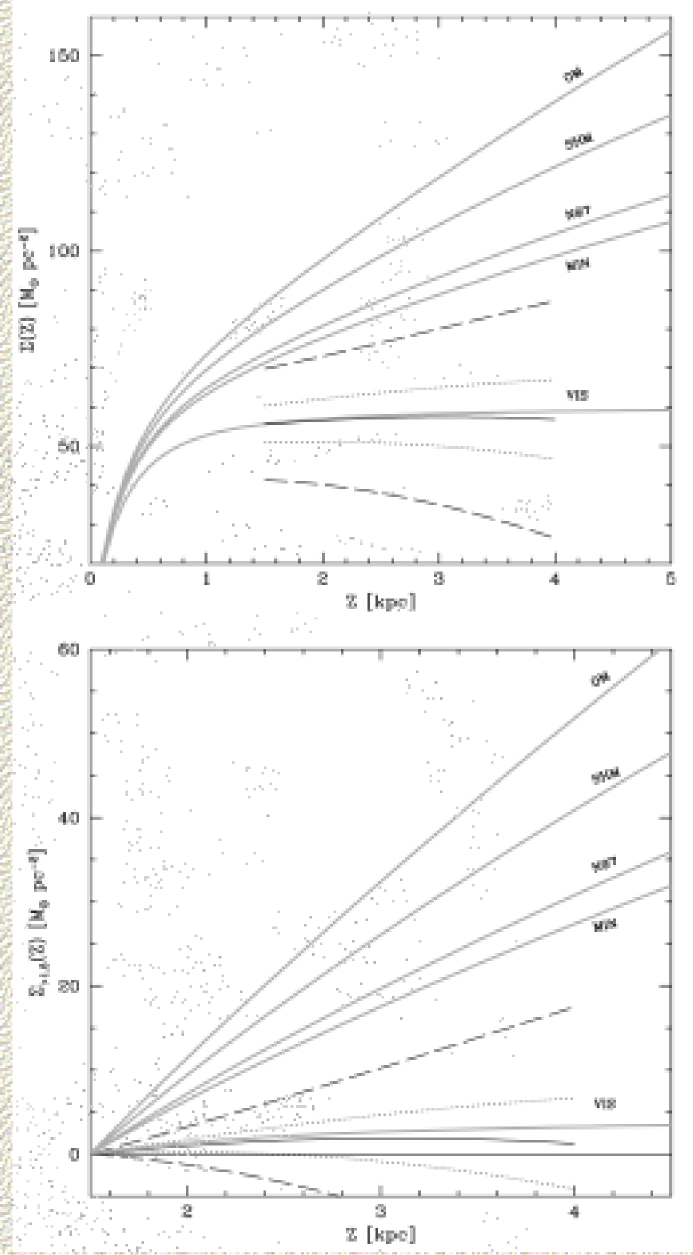
"Incremental" mass density: integration from $x$ to $z$

- no extrapolation

- insensitive to mass below $x$

*Results: no dark matter!*

$\rho_{DM,o} = 0 \pm 1 \ \mathrm{m}M_{\odot} \ \mathrm{pc}^{-3}$



Orthodox theory A, etherodox theory B

**If an observation matches the predictions of A rather than B:**

*large success of A, no doubts A is the right one, B is wrong, we must forget it*

**If an observation matches the predictions of B rather than A:**

*this means nothing, science is very complex, further tests are needed, probably there is a failure, maybe the data were not corrected for effect X, about which we know nothing, everything is so complex."*

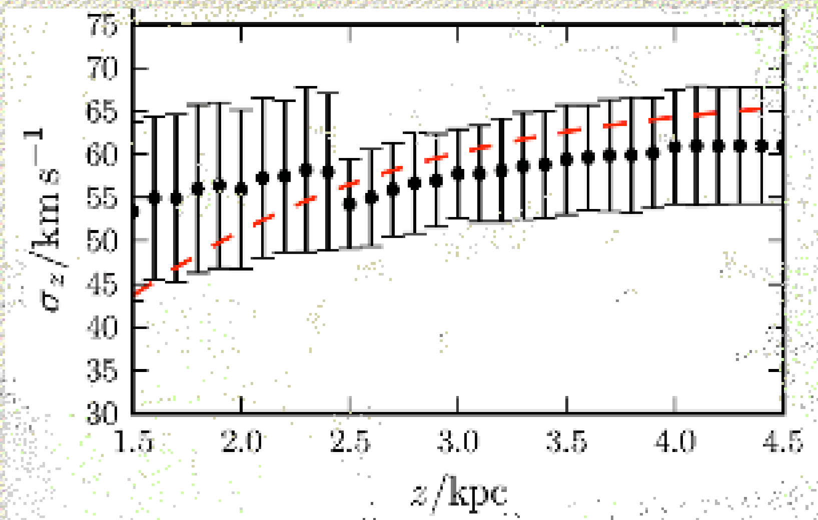
[López-Corredoira 2008]

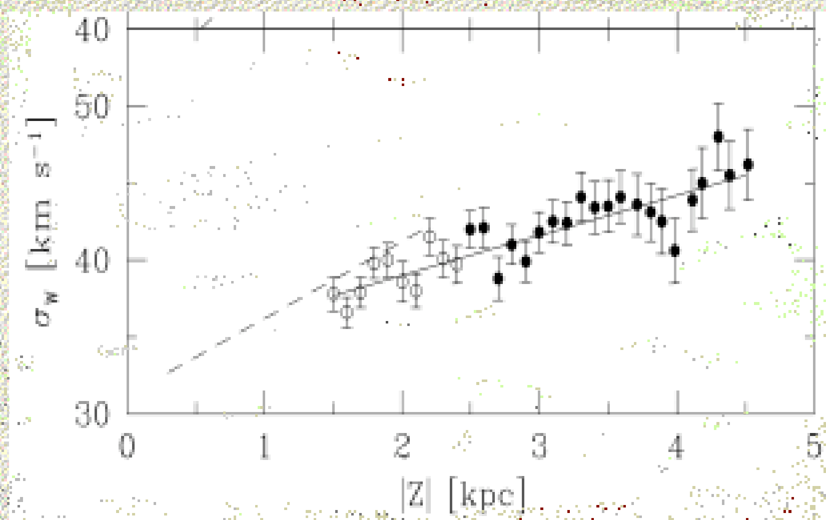
(Lopez-Corredoira 2009, ASPC, 409, 66)

# Sanders (2012, MNRAS, 425, 2228):

## the results are flawed because Moni Bidin et al. (2012) underestimated the dispersion gradients by a factor of three

1. Identical results with three data sets
2. Isotropic increase of dispersions does not increase noticeably the resulting mass
3. Simulations differ from measurements, indicating an underestimate of ≈15%



# Bovy & Tremaine (2012, ApJ, 759, 89, BT12):
## Assumption 9: "Rotational velocity constant with R at all z" is wrong

*Kill the radial term of Poisson equation, reduce to pseudo-1D formulation*

$$2\pi G\Sigma(Z) = -\int_0^Z \frac{1}{R}\frac{\partial(RF_R)}{\partial R}dz - F_z(Z) = -I_R(Z) - F_z(Z)$$

$\rho_{DM,o}$=7±2 $mM_o$ $pc^{-3}$

$I_R(Z)$ *small and negative:*

*- lower limit to mass density*

*- accurate within 20%*

$h_z$=0.7 kpc, $h_R$=2 kpc

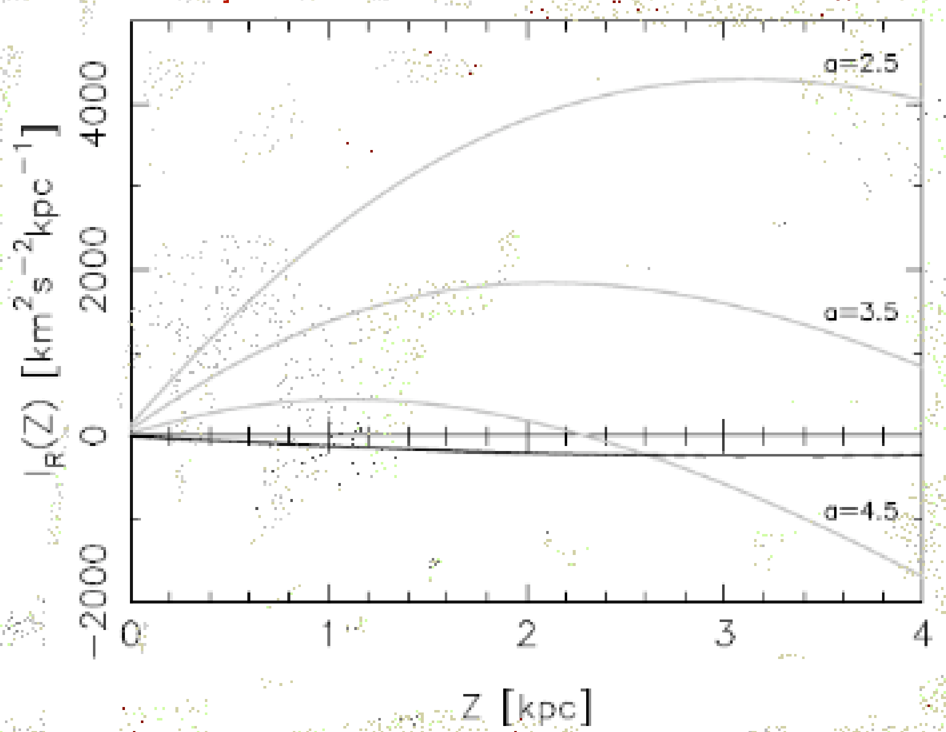
$\rho_{DM,o}$=8±3 $mM_o$ $pc^{-3}$

# The sign of $I_R(Z)$ depends on the mass distribution

$I_R(Z) > 0$ for a MN disk far from the center;

and for the triple-MN disk model of Flynn et al. (1996, MNRAS, 281, 1027)

$I_R(Z) < 0$ for a spherical component



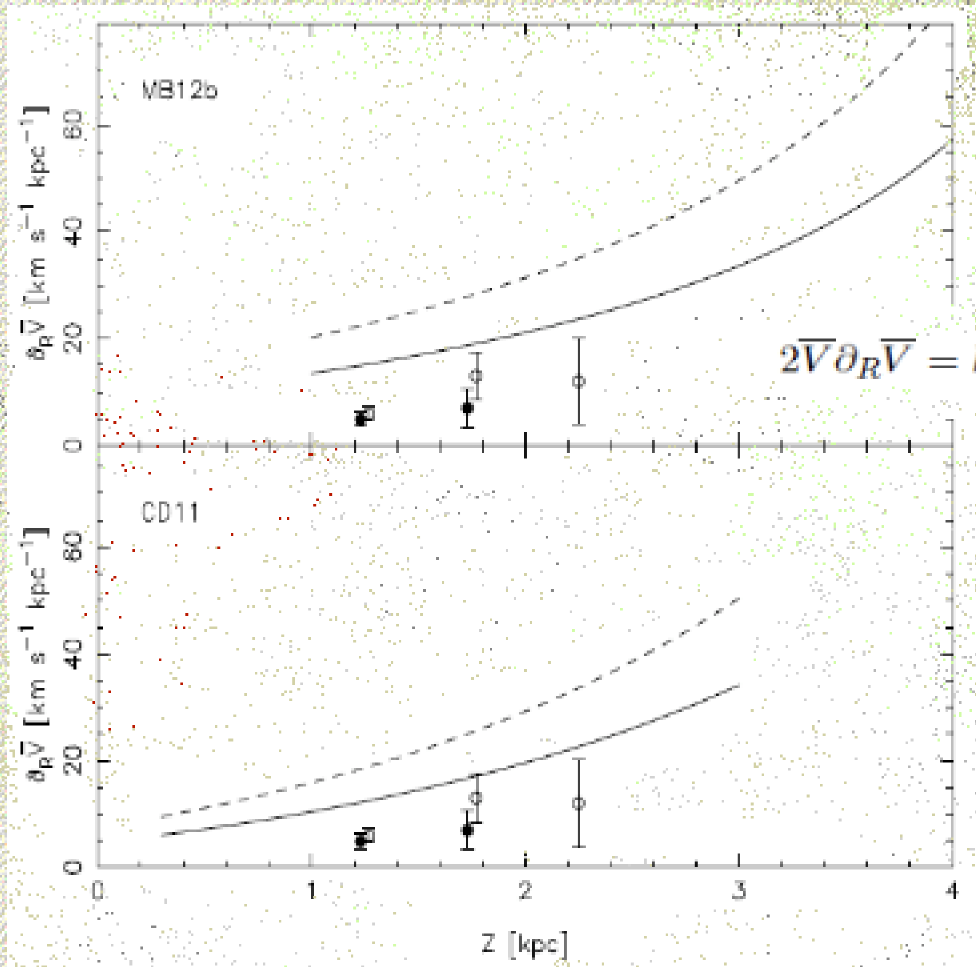

A NFW model dominates a MN disk in the range 0-4 kpc only if a>5 kpc

## The incidence of $I_R(Z)$ depends on the mass distribution

*A NFW halo with* $\rho_{DM,o}$**=7** $M_o$ $pc^{-3}$ *is required to have* $I_R/F_Z < 0.2$ *if the disk is a MN potential with* $a$=4 kpc

**Moreover:**

*If the total mass is overestiamted, the excess of visible mass is ascribed to dark matter, enhancing the overestiamte of* $\rho_{DM,o}$



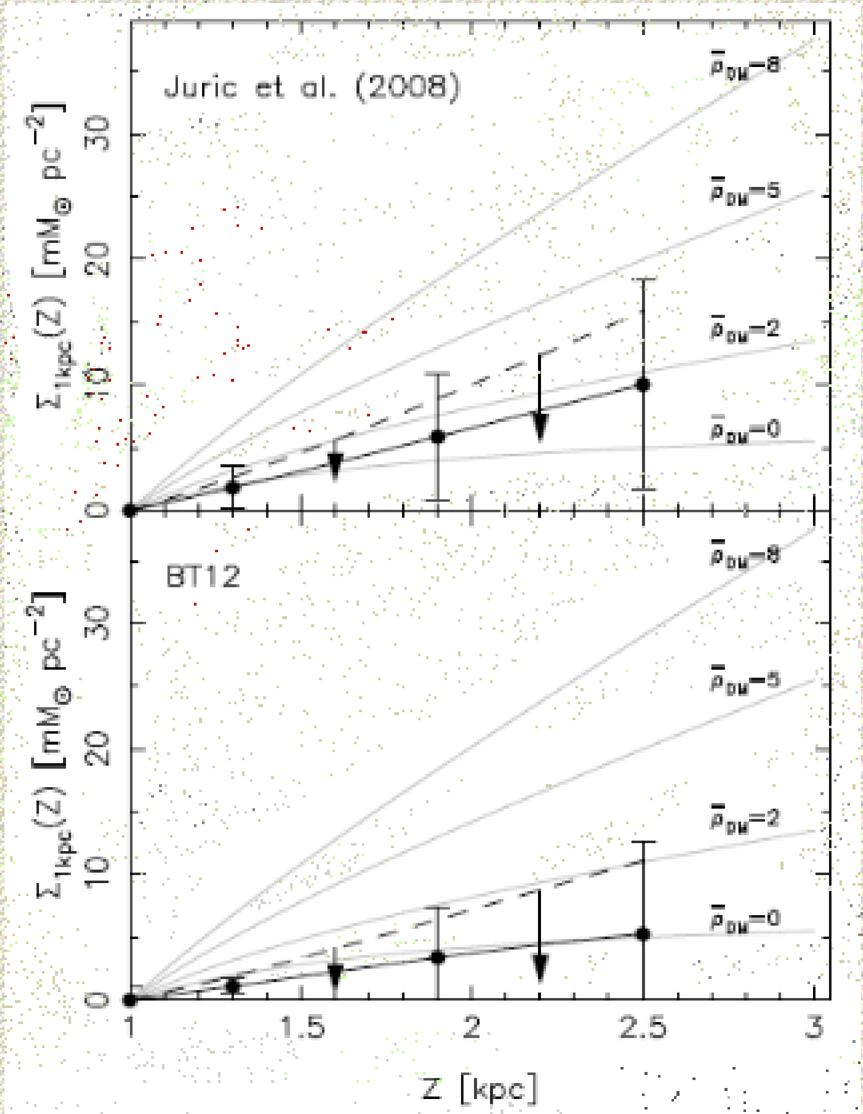

$I_R(Z)=0$ *requires a null integral*

$$2\overline{V}\partial_R\overline{V} = k_1\sigma_U^2 + \frac{\sigma_V^2}{h_\sigma} + \left(\frac{R}{h_\sigma}-1\right)\left(\frac{\overline{UW}}{h_Z} - \frac{\partial\overline{UW}}{\partial Z}\right)$$

*Very peculiar kinematics, excluded by observations of external galaxies and the Milky Way*

*BT12 results are flawed by an assumption which casts wrong predictions on the thick disk rotation, resulting in a mass overestimate*

# Moni Bidin et al. (2012) revisited:




$\rho_{DM}<3.6\pm3.0$ $mM_\odot$ $pc^{-3}$

$\rho_{DM}=2\pm3$ $mM_\odot$ $pc^{-3}$

$\rho_{DM}<2\pm3$ $mM_\odot$ $pc^{-3}$

$\rho_{DM}=0\pm2$ $mM_\odot$ $pc^{-3}$

# Conclusions

- Lack of dark matter confirmed: BT12 is not the solution

- Why 3D calculations return no mass excess?
- Are the results repeatable?
- Is the integration from **z=0** reliable?