



GARRA Group



Instituto Argentino de
Radioastronomía



ISSN: 1853-5461

Gravitational waves: history, detection, and prospects

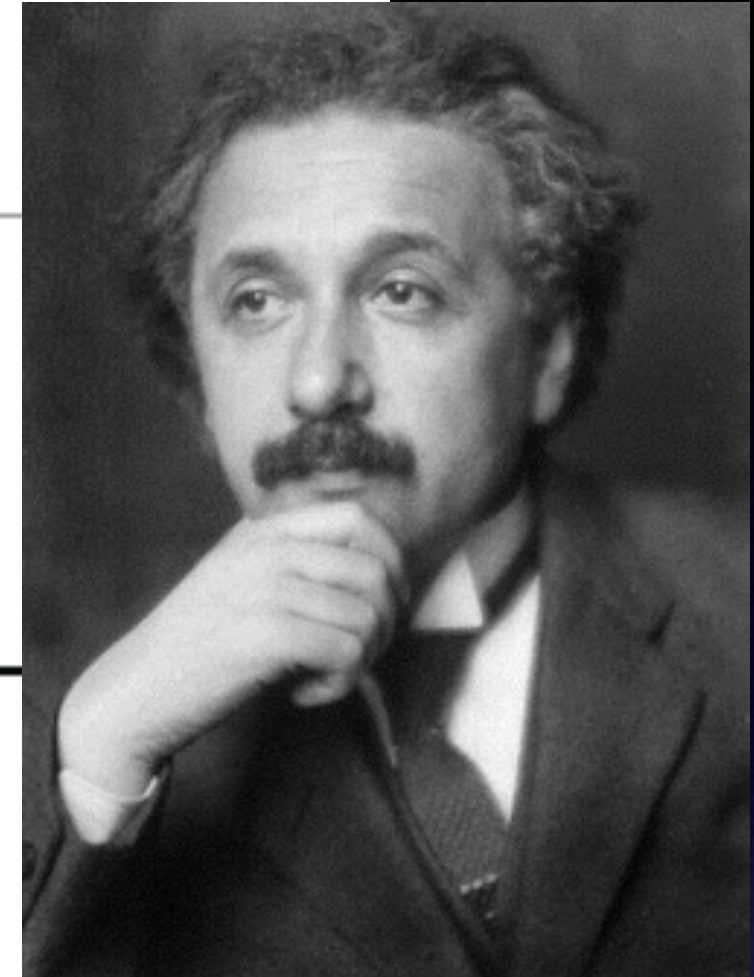
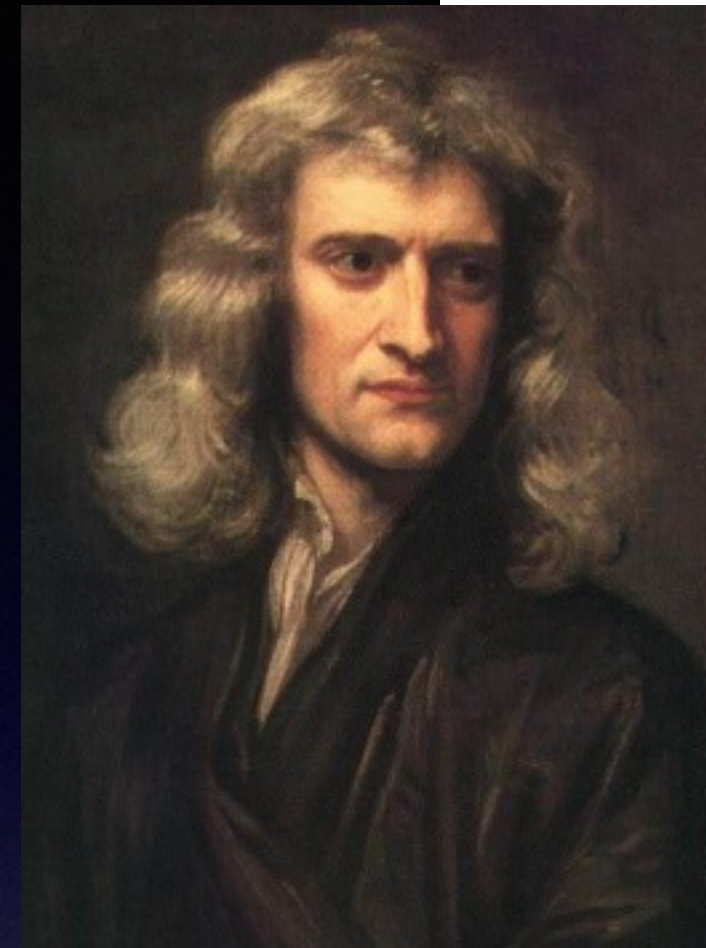
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IAR-CONICET/FCAG-UNLP, Argentina

FoF, April 1st, 2016

OAC, Córdoba

Newtonian vs General Relativistic gravity



Newtonian field equations

GR field equations

$$\nabla^2 \Phi = 4 \pi G \rho$$

$$G^{ab} = \frac{8\pi G}{c^4} T^{ab}$$

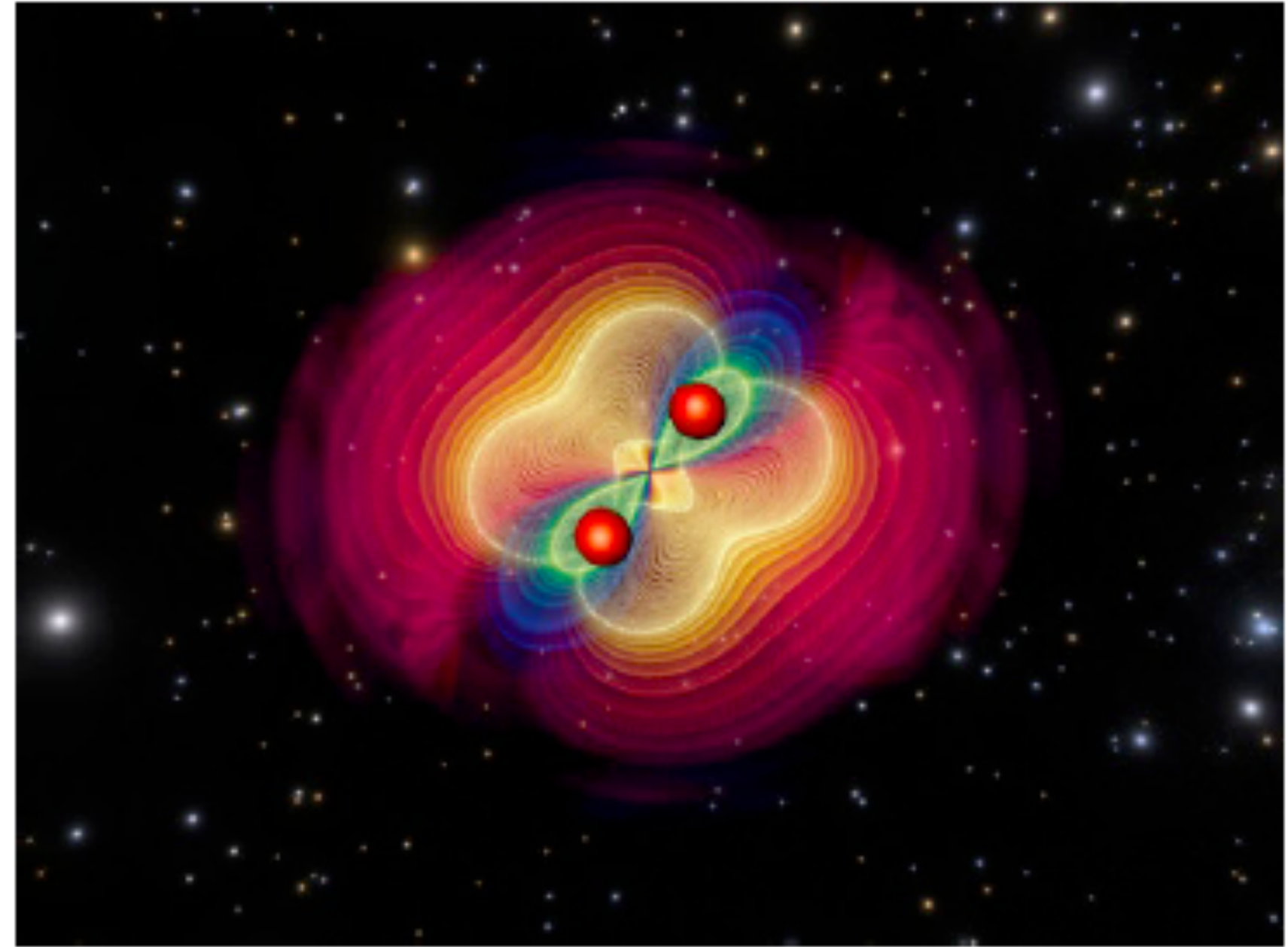
Source: mass density

Source: **energy-momentum tensor**
(includes mass densities/currents)

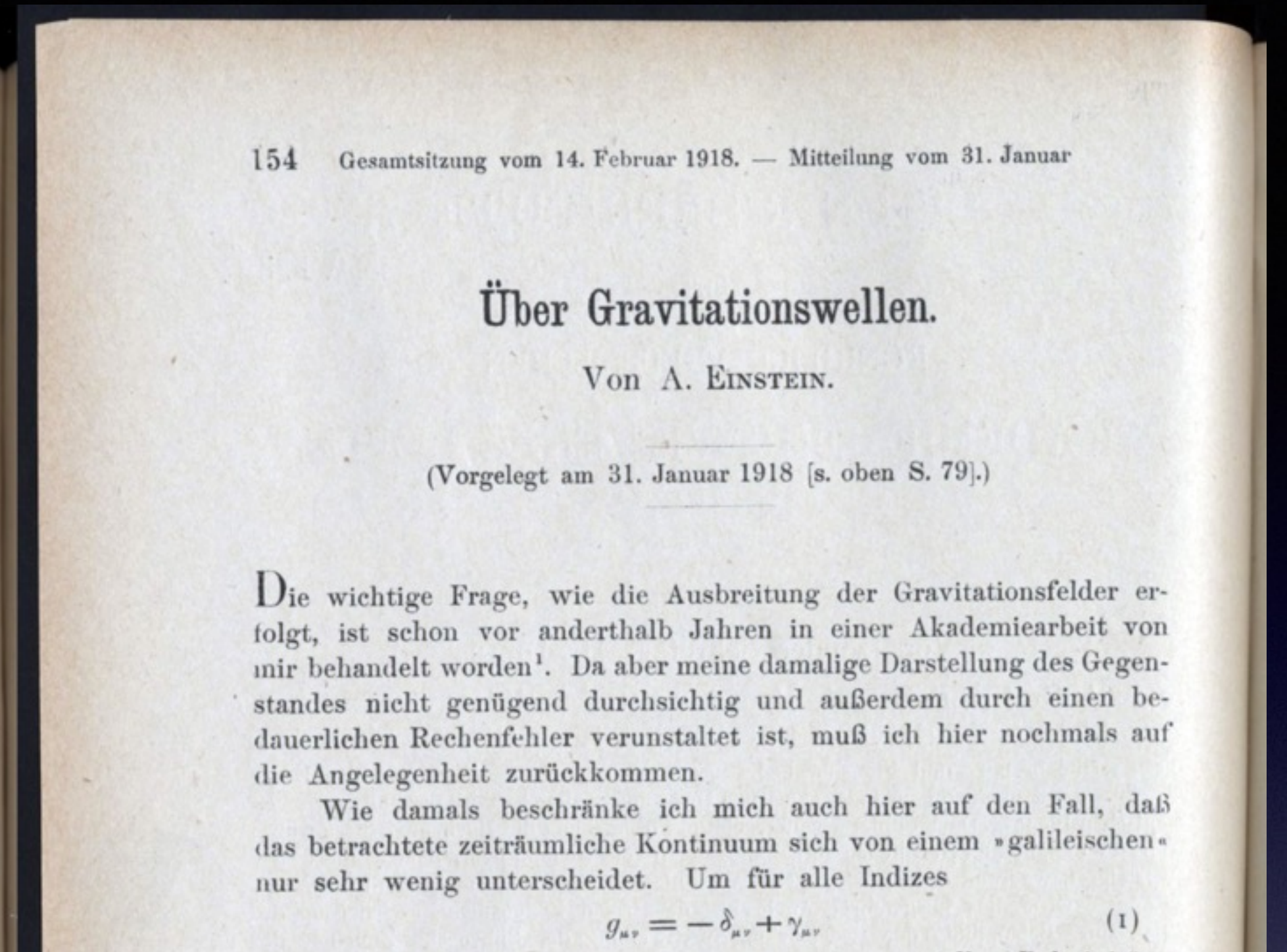
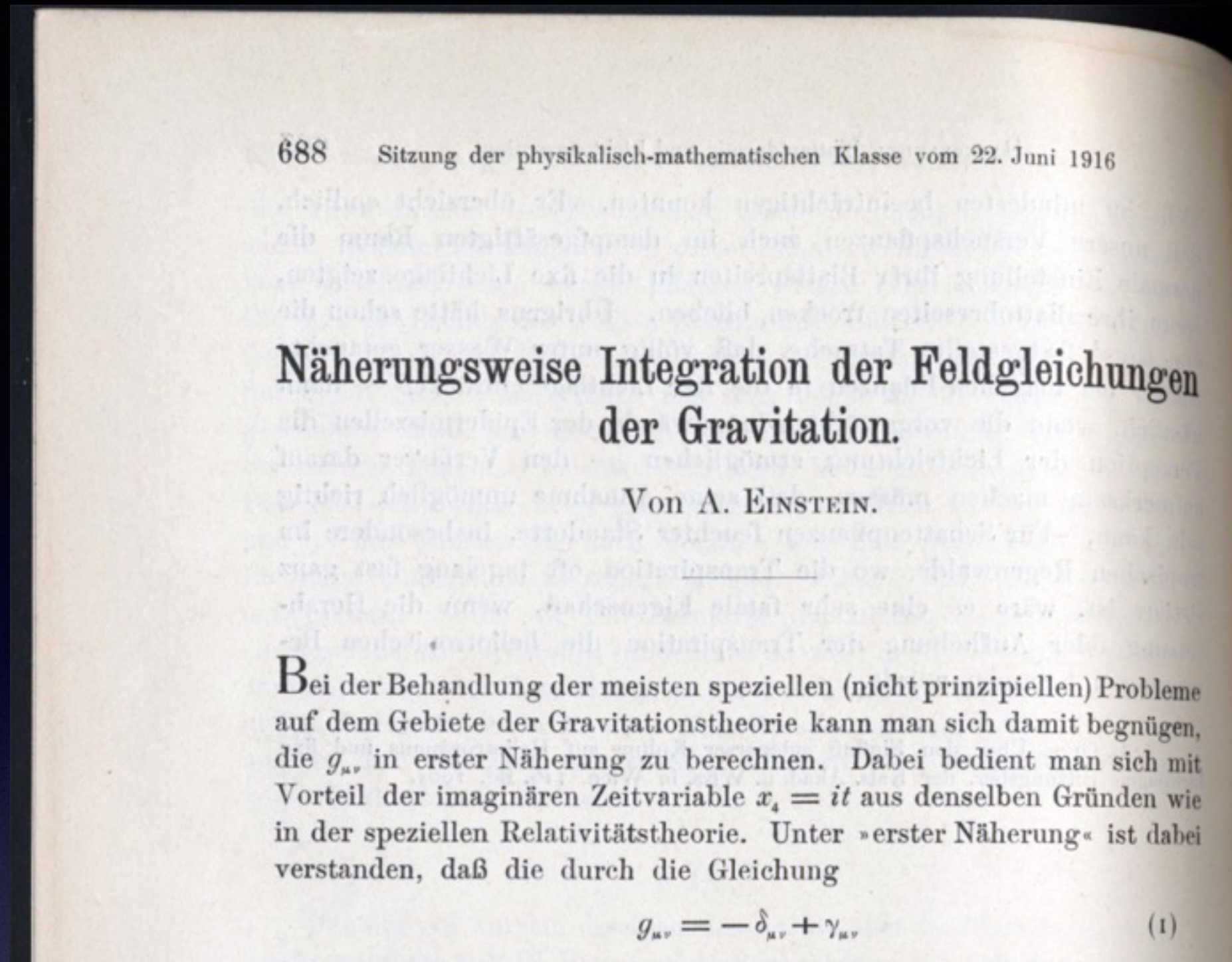
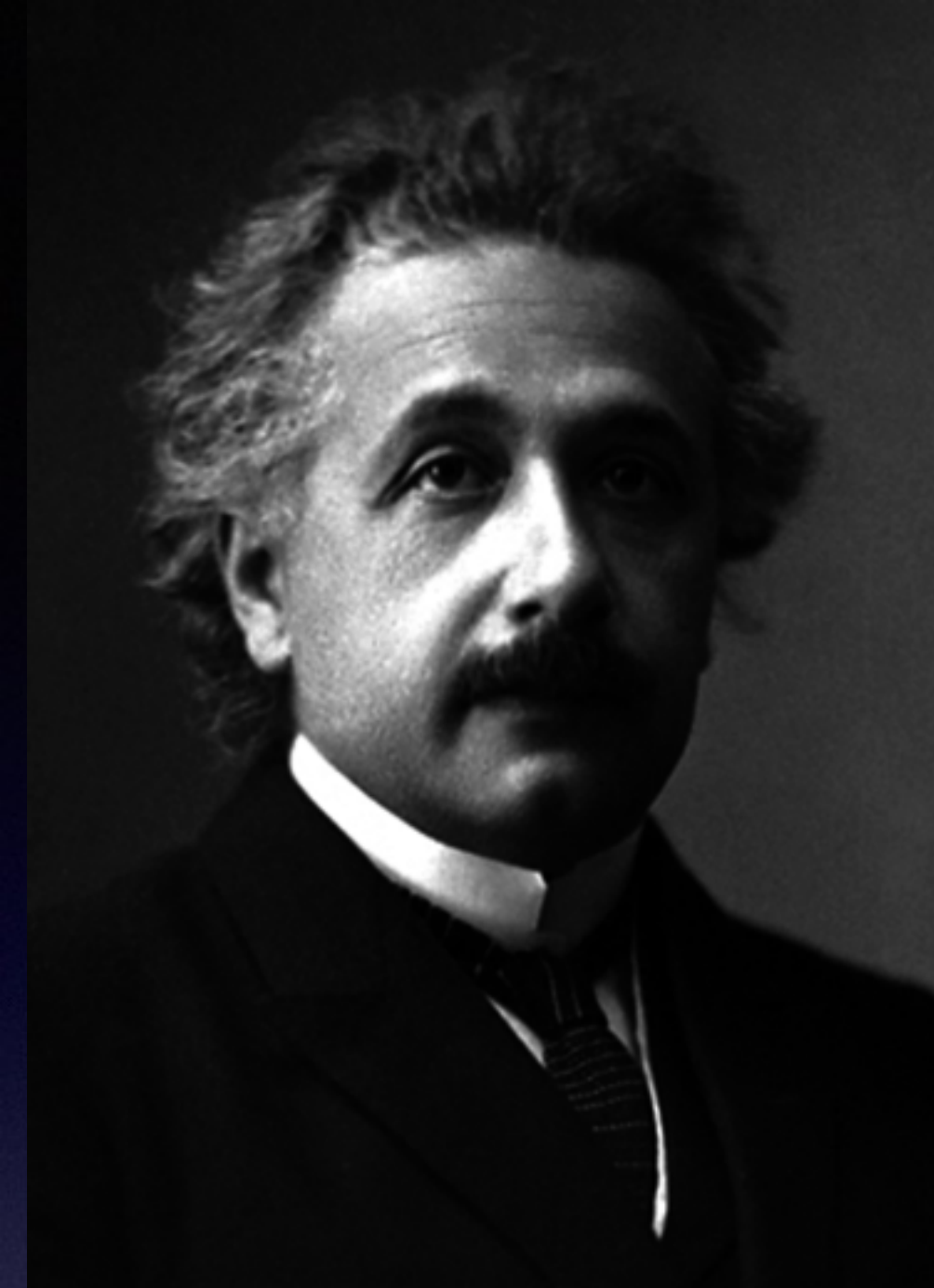
Gravitational field: scalar Φ

Gravitational field: **metric tensor** g_{ab}

- **Electromagnetism:** accelerating charges produce EM radiation.



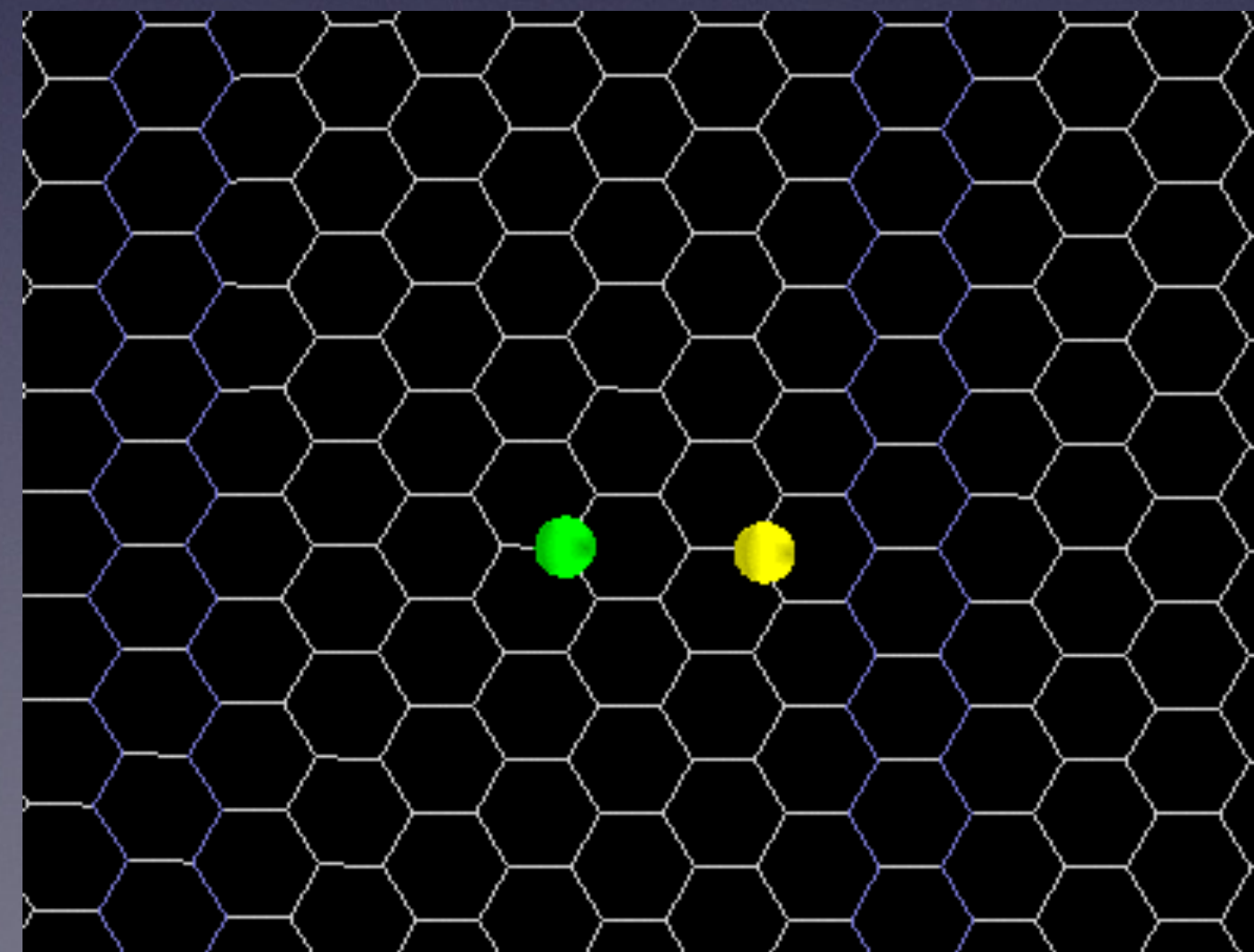
- **Gravitation:** accelerating masses produce gravitational radiation.
(another hint: gravity has finite speed.)



Two seminal papers

1916

1918



GWs in linear gravity

- We consider **weak** gravitational fields:

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h_{\mu\nu}^2)$$

↑
flat Minkowski metric

- The GR field equations in vacuum reduce to the standard **wave equation**:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) h^{\mu\nu} = \square h^{\mu\nu} = 0$$

- Comment: GR gravity like electromagnetism has a “**gauge**” freedom associated with the choice of coordinate system. The above equation applies in the so-called “**transverse-traceless (TT)**” gauge where

$$h_{0\mu} = 0, \quad h^\mu{}_\mu = 0$$

Wave solutions

- Solving the previous wave equation in weak gravity is easy. The solutions represent “plane waves”:

$$h_{\mu\nu} = A_{\mu\nu} e^{ik_a x^a}$$

↑
wave-vector

- Basic properties: $A_{\mu\nu} k^\mu = 0$, $k_a k^a = 0$

↑
transverse waves

↑
null vector = propagation along light rays

- Amplitude: $A^{\mu\nu} = h_+ e_+^{\mu\nu} + h_\times e_\times^{\mu\nu}$

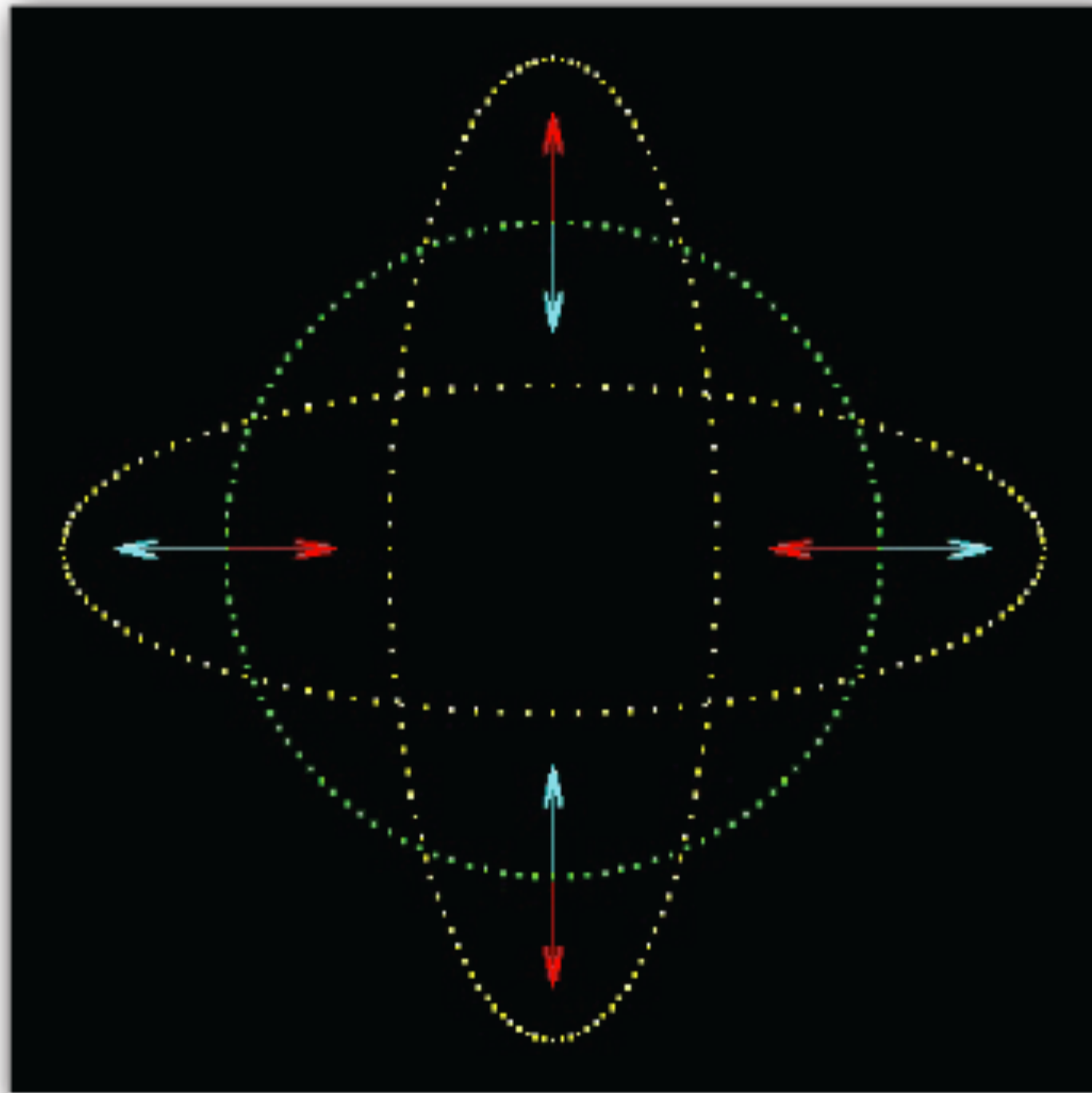
↑
two polarizations

$$e_+^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

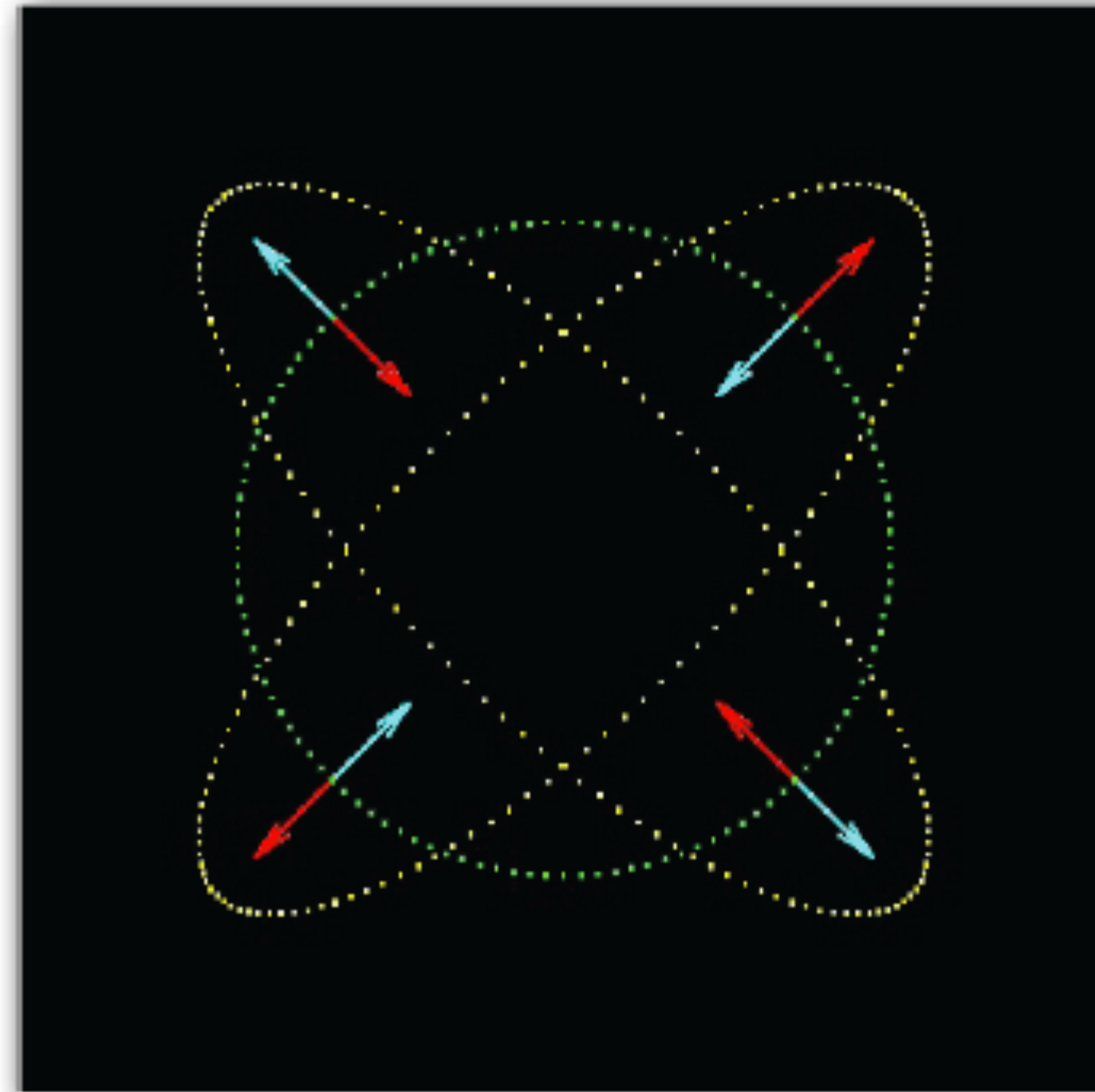
$$e_\times^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

GWs: polarization

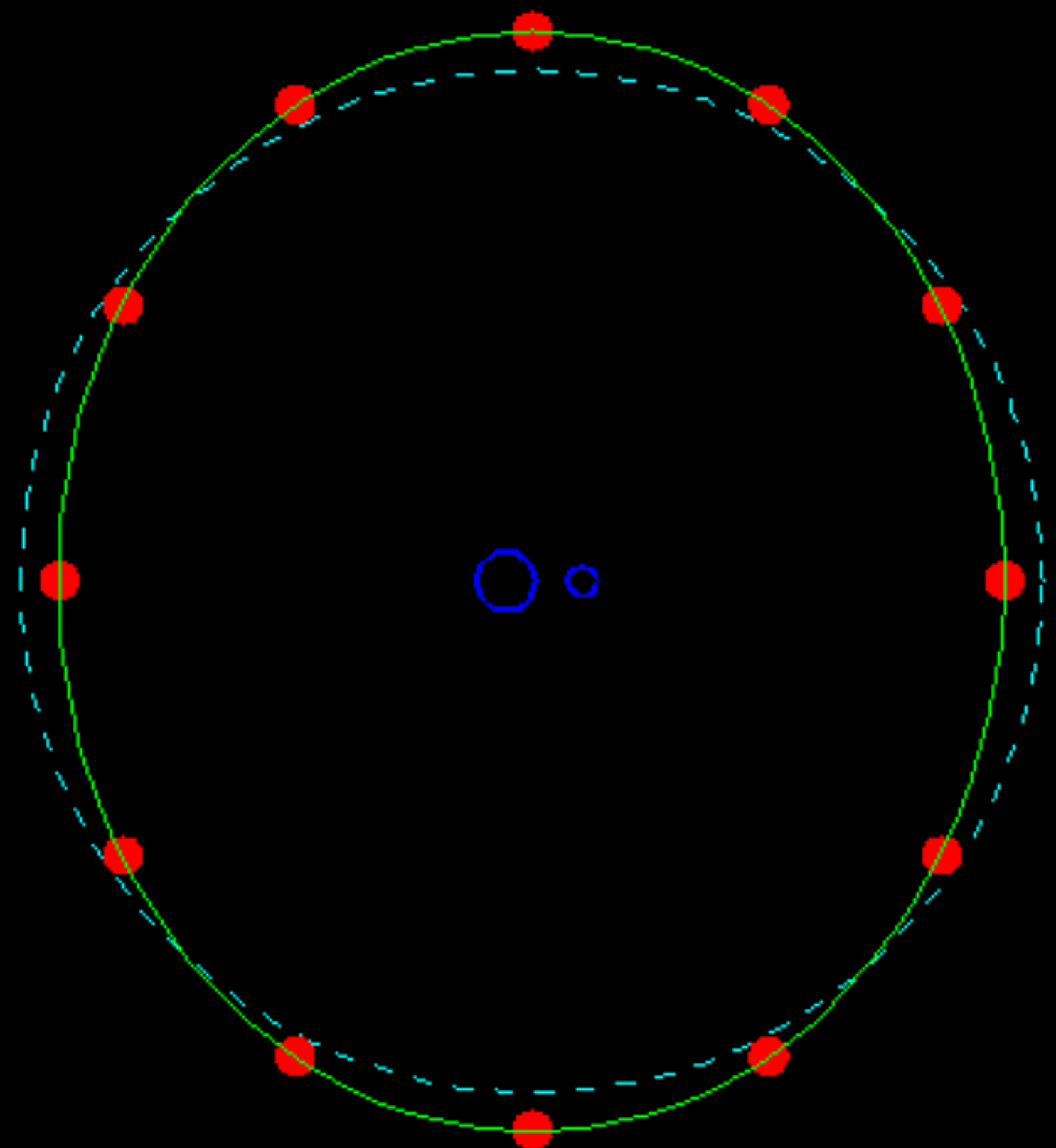
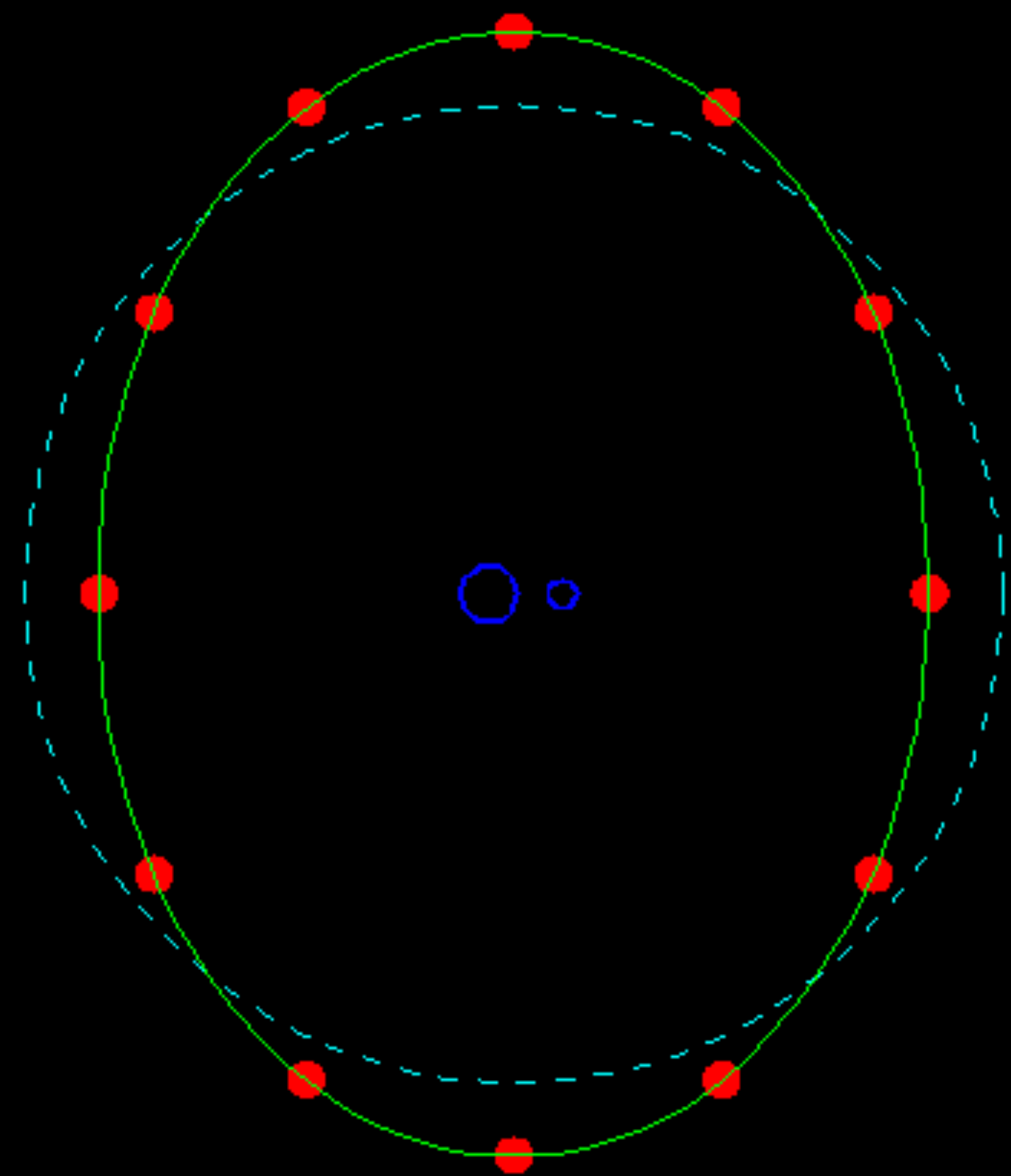
- GWs come in two polarizations:



“+” polarization



“x” polarization



GWs: more properties

- EM waves: at lowest order the radiation can be emitted by a dipole source ($l=1$). Monopolar radiation is forbidden as a result of charge conservation.
- GWs: the lowest allowed multipole is the **quadrupole** ($l=2$). The monopole is forbidden as a result of mass conservation. Similarly, dipole radiation is absent as a result of momentum conservation.
- GWs represents propagating “ripples in spacetime” or, more accurately, a **propagating curvature perturbation**. The perturbed curvature (Riemann tensor) is given by (in the TT gauge):

$$R_{j0k0}^{\text{TT}} = -\frac{1}{2} \partial_t^2 h_{jk}^{\text{TT}}, \quad j, k = 1, 2, 3$$

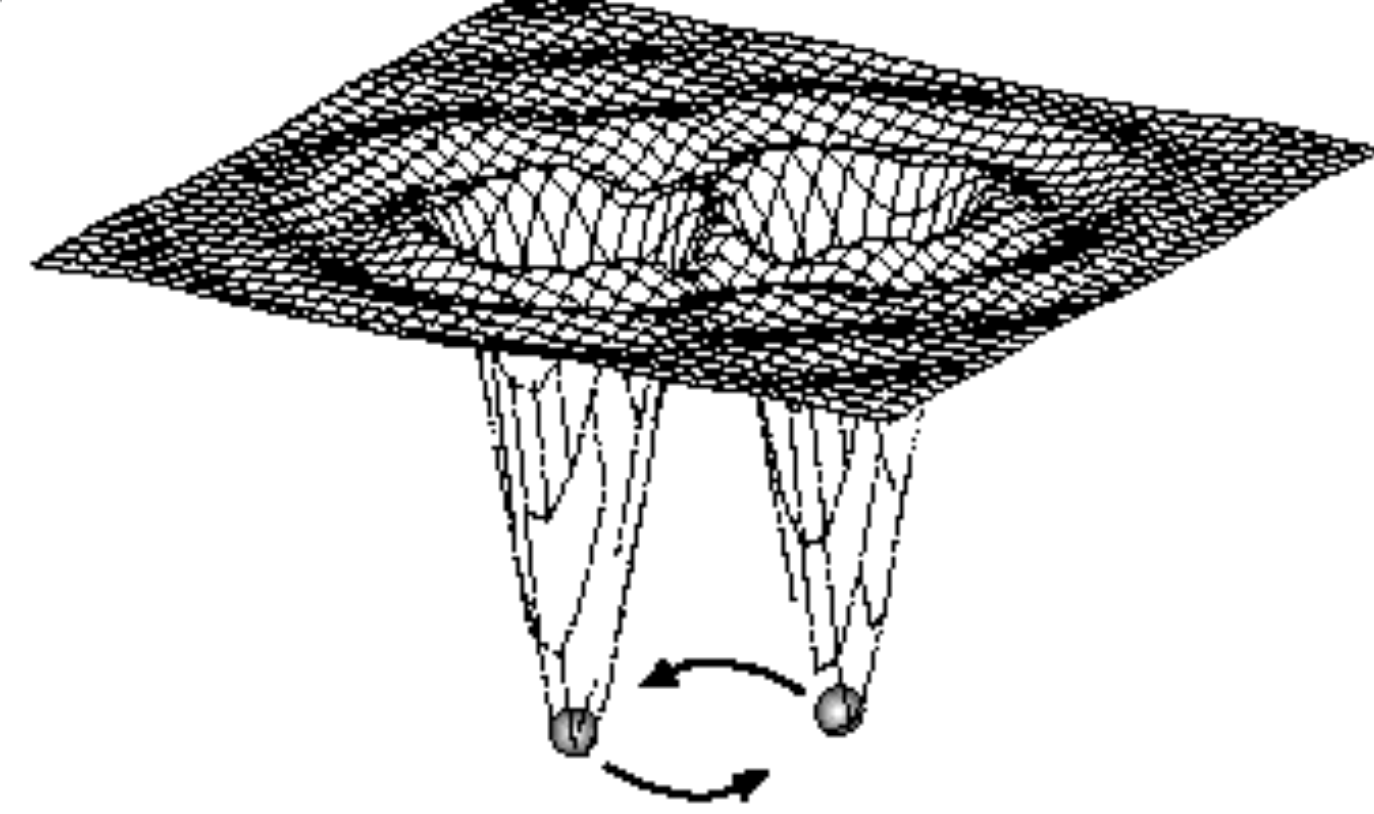
GWs and curvature

- As we mentioned, GWs represent a fluctuating curvature field.
- Their effect on test particles is of tidal nature.
- Equation of **geodesic deviation** (in weak gravity):

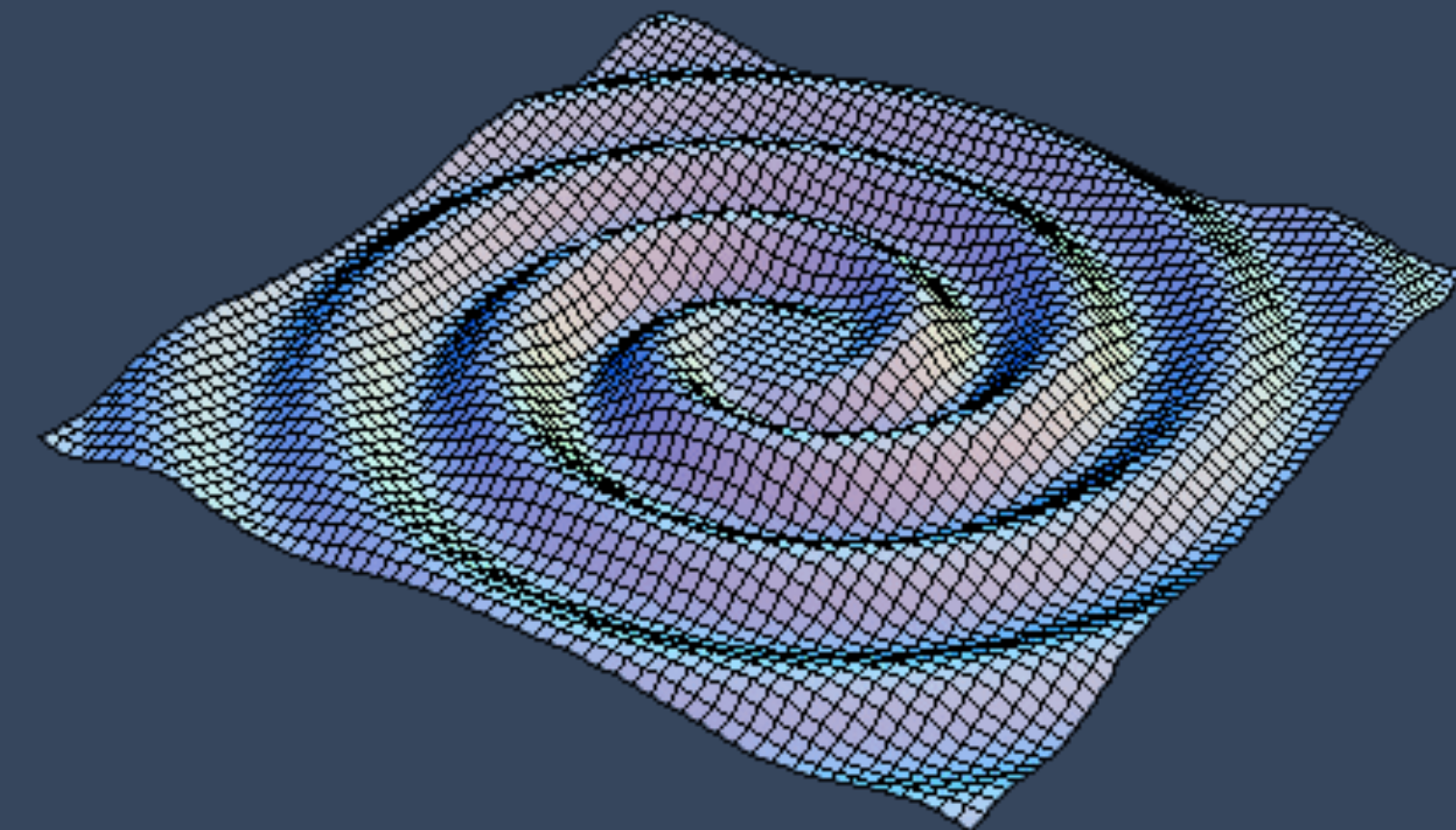
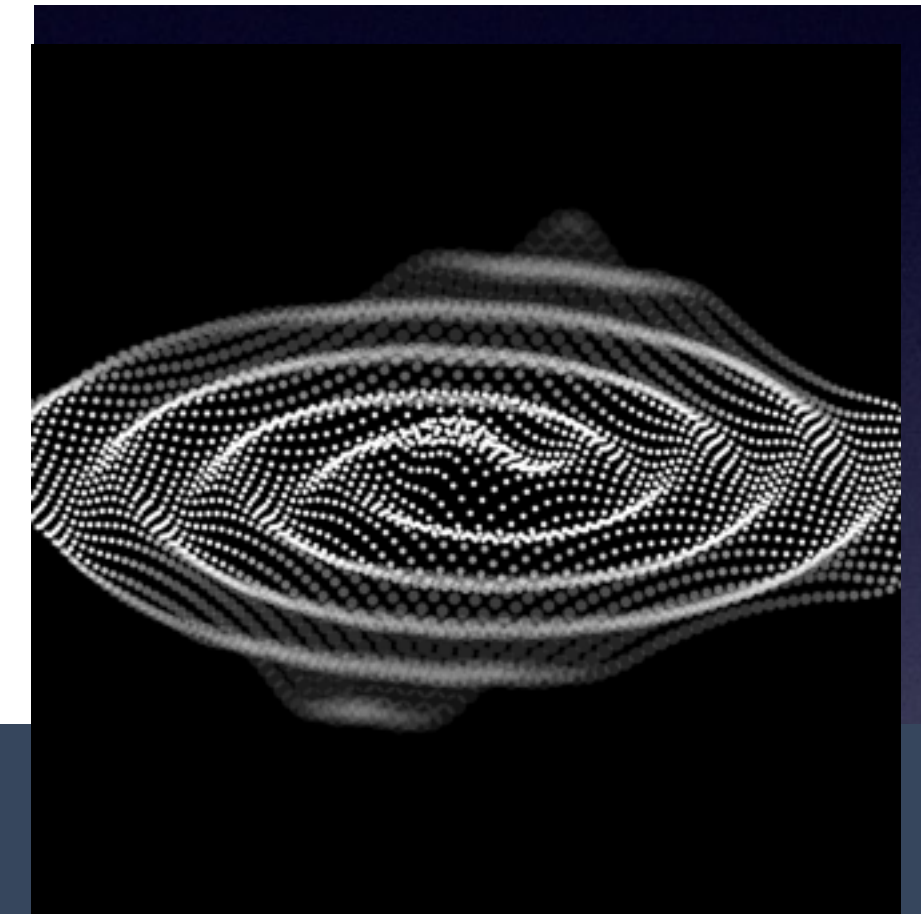
$$\frac{d^2 \xi^k}{dt^2} = -R_{0j0}^k{}^{\text{TT}} \xi^j$$

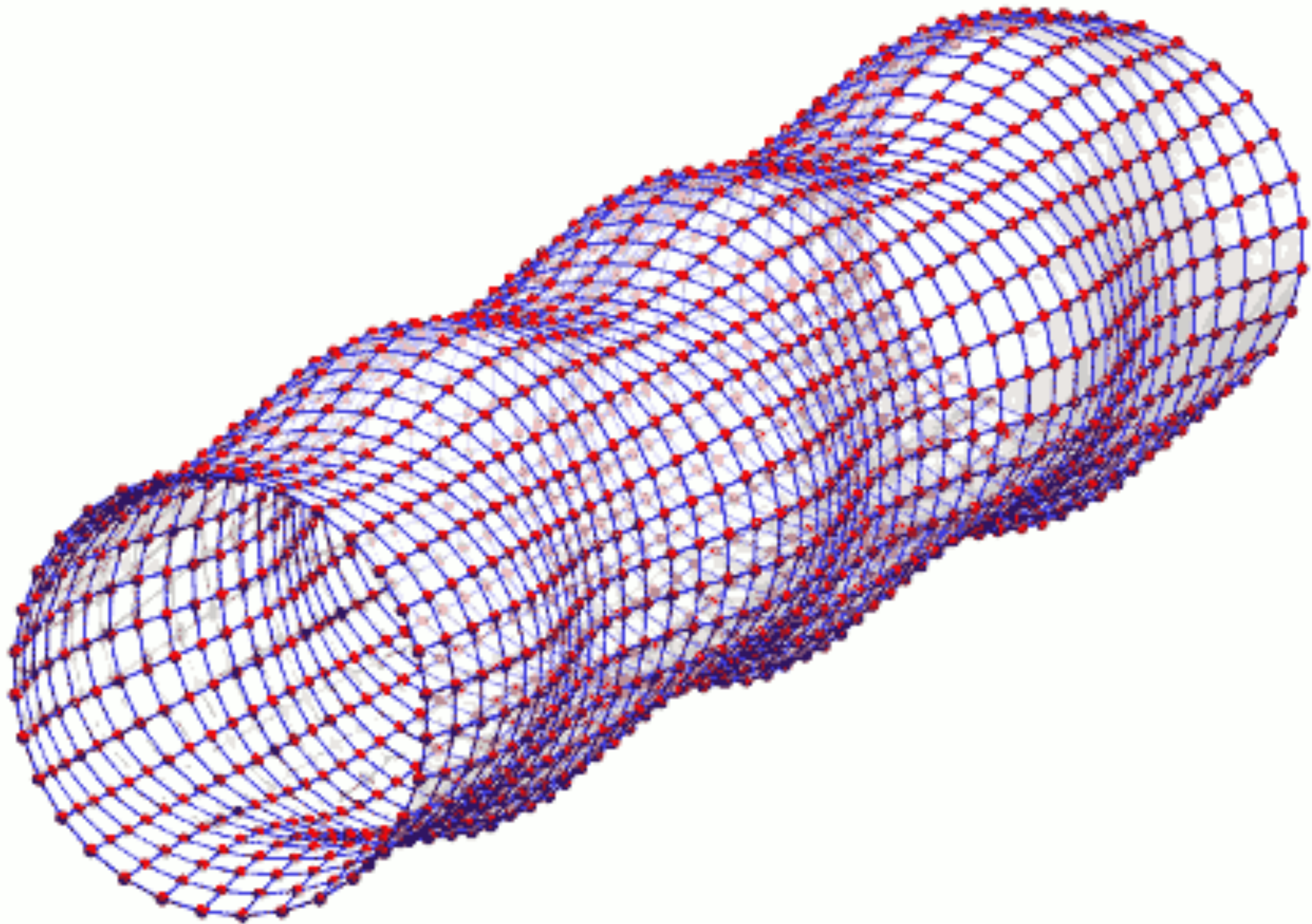
distance between geodesics (test particles)

- Newtonian limit: $R_{k0j0}^{\text{TT}} \approx \frac{\partial^2 \Phi}{\partial x^k \partial x^j}$ ← Newtonian grav. potential



A binary system of compact massive objects rapidly orbiting each other produces ripples in spacetime.





+ waves

GWs vs EM waves

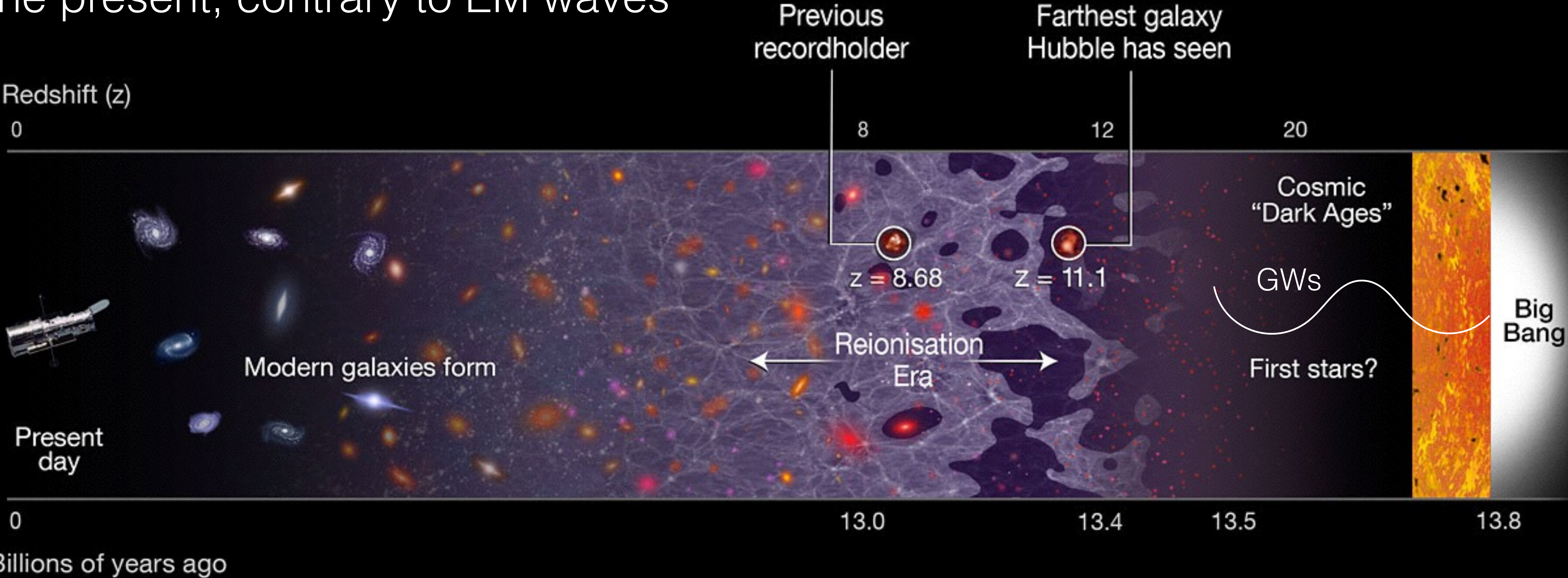
- Similarities:

- ✓ Propagation with the speed of light.
- ✓ Amplitude decreases as $\sim 1/r$.
- ✓ Frequency redshift (Doppler, gravitational, cosmological).

- Differences:

- ✓ GWs propagate through matter with little interaction. Hard to detect, but they carry uncontaminated information about their sources.
- ✓ Strong GWs are generated by bulk (coherent) motion. They require strong gravity/high velocities (compact objects like black holes and neutron star).
- ✓ EM waves originate from small-scale, incoherent motion of charged particles. They are subject to “environmental” contamination (interstellar absorption etc.).

GW can propagate from the inflationary period, if it existed, to the present, contrary to EM waves

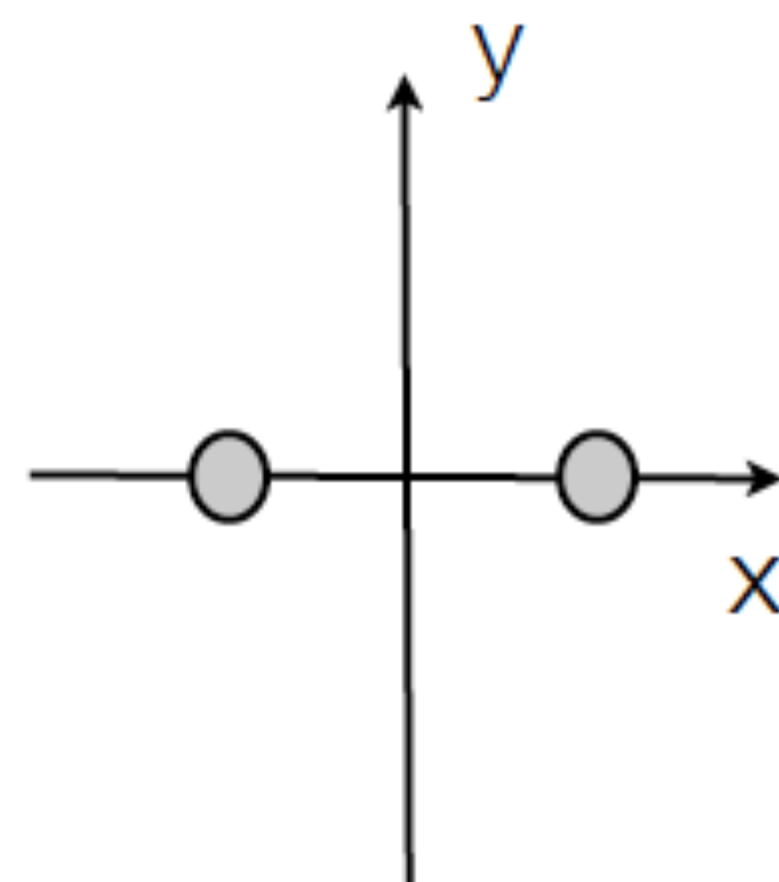


Effect on test particles

- We consider a pair of test particles on the cartesian axis Ox at distances $\pm x_0$ from the origin and we assume a GW traveling in the z -direction.
- Their distance will be given by the relation:

$$\begin{aligned} dl^2 &= g_{\mu\nu} dx^\mu dx^\nu = \dots = -g_{11} dx^2 = \\ &= (1 - h_{11})(2x_0)^2 = [1 - h_+ \cos(\omega t)] (2x_0)^2 \end{aligned}$$

$$dl \approx \left[1 - \frac{1}{2} h_+ \cos(\omega t) \right] (2x_0)$$



The quadrupole formula

- Einstein (1918) derived the quadrupole formula for gravitational radiation by solving the linearized field equations with a source term:

$$\square h^{\mu\nu}(t, \vec{x}) = -\kappa T^{\mu\nu}(t, \vec{x}) \longrightarrow h^{\mu\nu} = -\frac{\kappa}{4\pi} \int_V d^3x' \frac{T^{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}$$

- This solution suggests that the wave amplitude is proportional to the **second time derivative of the quadrupole** moment of the source:

$$h^{\mu\nu} = \frac{2G}{r c^4} \ddot{Q}_{\text{TT}}^{\mu\nu}(t - r/c) \qquad Q_{\text{TT}}^{\mu\nu} = \int d^3x \rho \left(x^\mu x^\nu - \frac{1}{3} \delta^{\mu\nu} r^2 \right)$$

(quadrupole moment in the “TT gauge” and at the retarded time $t-r/c$)

- This result is quite accurate for all sources, **as long as the wavelength is much longer than the source size R .**

GW luminosity

- **GWs carry energy.** The stress-energy carried by GWs cannot be localized within a wavelength. Instead, one can say that a certain amount of stress-energy is contained in a region of the space which extends over several wavelengths. The **stress-energy tensor** can be written as:

$$T_{\mu\nu}^{\text{GW}} = \frac{c^4}{32\pi G} \langle \partial_\mu h_{ij}^{\text{TT}} \partial_\nu h_{\text{TT}}^{ij} \rangle$$

- Using the previous quadrupole formula we obtain the **GW luminosity**:

$$L_{\text{GW}} = \frac{dE_{\text{GW}}}{dt} = \int dA T_{0j}^{\text{GW}} \hat{n}^j \quad \longrightarrow \quad L_{\text{GW}} = \frac{1}{5} \frac{G}{c^5} \langle \ddot{Q}_{\mu\nu}^{\text{TT}} \ddot{Q}_{\text{TT}}^{\mu\nu} \rangle$$

Basic estimates

- The luminosity of GWs from a given source is approximately:

$$L_{\text{GW}} \sim \frac{G^4}{c^5} \left(\frac{M}{R}\right)^5 \sim \frac{G}{c^5} \left(\frac{M}{R}\right)^2 v^6 \sim \frac{c^5}{G} \left(\frac{R_{\text{Sch}}}{R}\right)^2 \left(\frac{v}{c}\right)^6$$

where $R_{\text{Sch}} = 2GM/c^2$ is the Schwarzschild radius of the source. It is obvious that the maximum GW luminosity can be achieved if $R \sim R_{\text{Sch}}$ and $v \sim c$. That is, the source needs to be compact and relativistic.

- Using the above order-of-magnitude estimates, we can get a rough estimate of the amplitude of GWs at a distance r from the source:

$$h \sim \frac{G}{c^4} \frac{E_{\text{ns}}}{r} \sim \frac{G}{c^4} \frac{\epsilon E_{\text{kin}}}{r}$$

$\epsilon =$ the kinetic energy fraction that is able to produce GWs.

Basic estimates

- Another estimate for the GW amplitude can be derived from the flux formula

$$F_{\text{GW}} = \frac{L_{\text{GW}}}{4\pi r^2} = \frac{c^3}{16\pi G} |\partial_t h|^2$$

- We obtain:

$$h \approx 10^{-22} \left(\frac{E_{\text{GW}}}{10^{-4} M_{\odot}} \right)^{1/2} \left(\frac{1 \text{ kHz}}{f_{\text{GW}}} \right) \left(\frac{\tau}{1 \text{ ms}} \right)^{-1/2} \left(\frac{15 \text{ Mpc}}{r} \right)$$

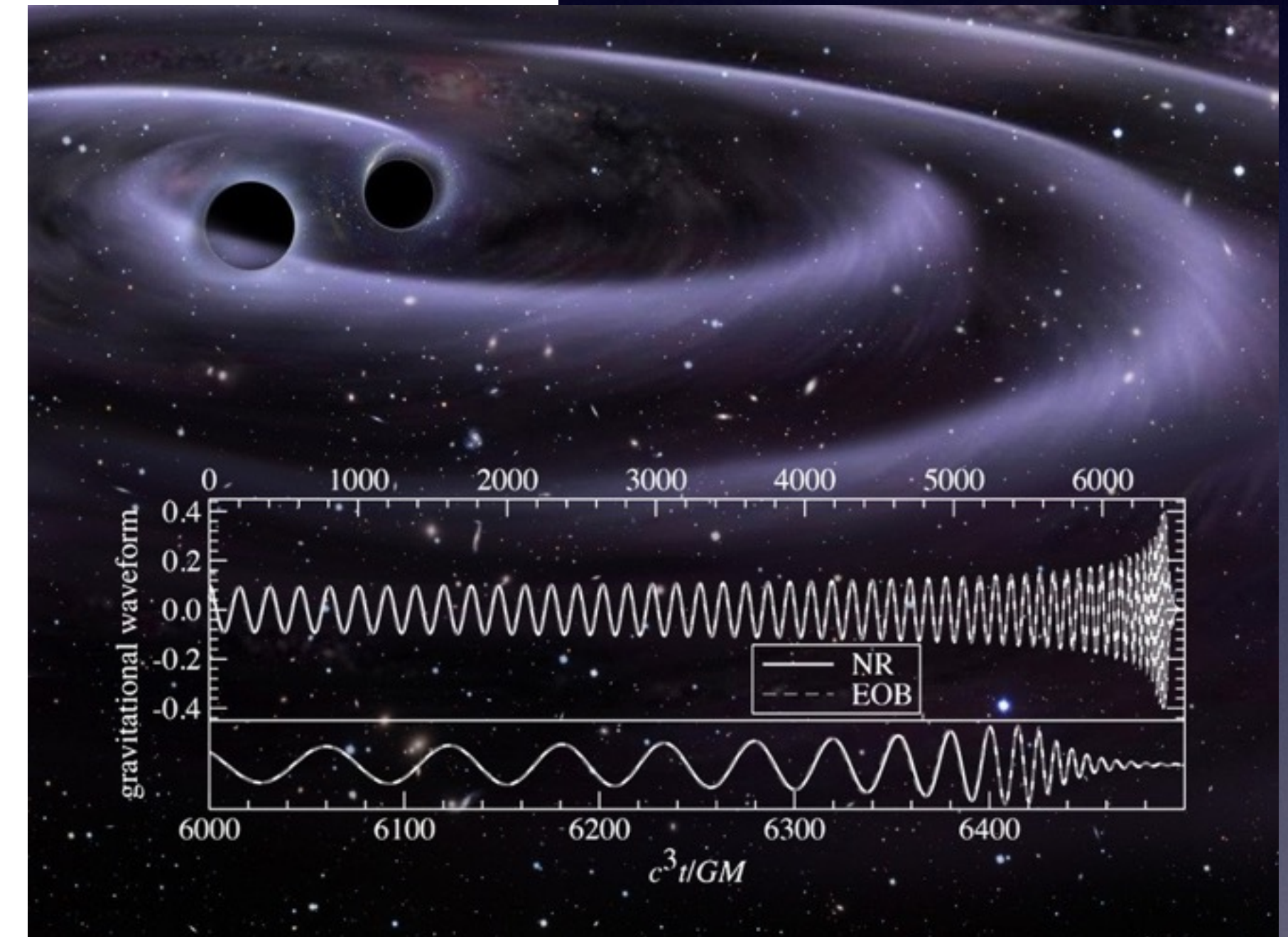
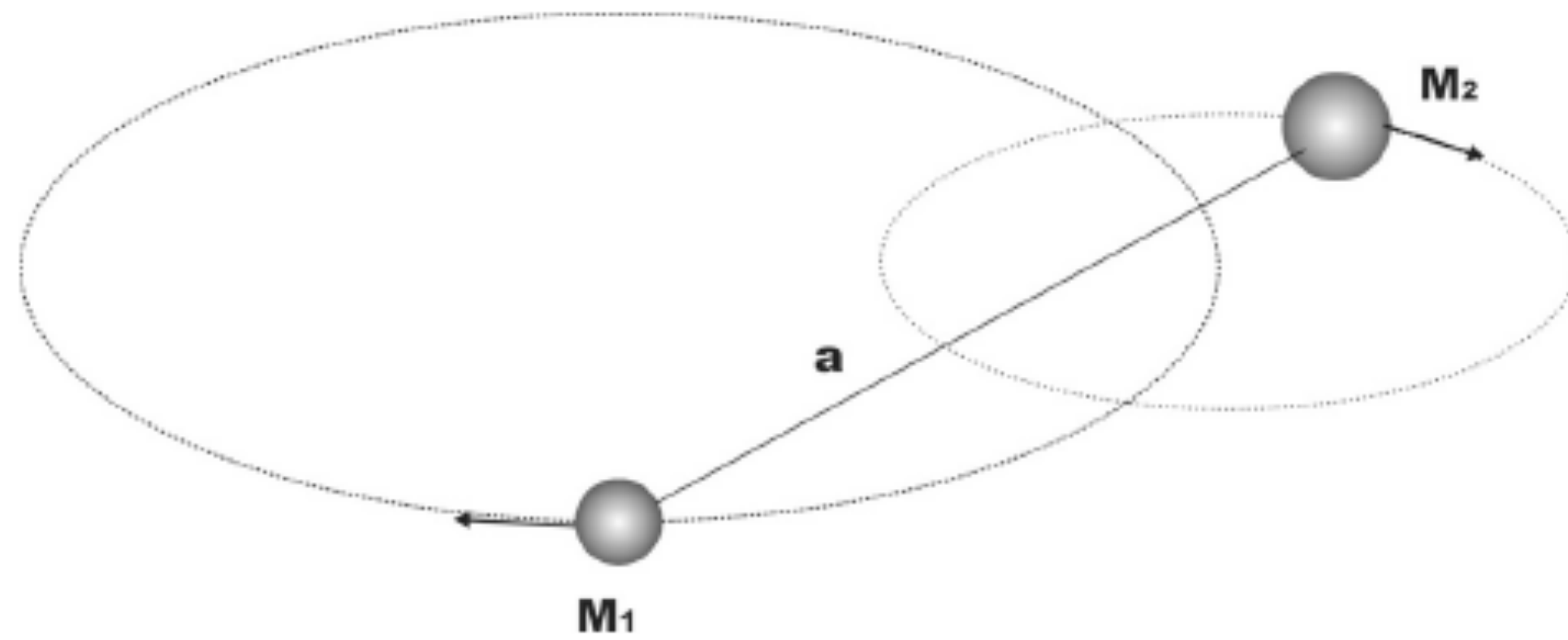
for example, this formula could describe the GW strain from a supernova explosion at the Virgo cluster during which the energy E_{GW} is released in GWs at a frequency of **1 kHz**, and with signal duration of the order of **1 ms**.

- This is why **GWs are hard to detect**: for a GW detector with arm length of $l = 4 \text{ km}$ we are looking for changes in the arm-length of the order of

$$\Delta l = hl = 4 \times 10^{-17} \text{ cm} \quad !!$$

GW emission from a binary system (I)

- The binary consists of the two bodies M_1 and M_2 at distances a_1 and a_2 from the center of mass. The orbits are circular and lie on the x-y plane. The orbital angular frequency is Ω .



- We also define: $a = a_1 + a_2$, $\mu = M_1 M_2 / M$, $M = M_1 + M_2$

GW emission from a binary system (II)

- The only non-vanishing components of the quadrupole tensor are :

$$Q_{xx} = -Q_{yy} = (a_1^2 M_1 + a_2^2 M_2) \cos^2(\Omega t) = \frac{1}{2} \mu a^2 \cos(2\Omega t)$$

$$Q_{xy} = Q_{yx} = \frac{1}{2} \mu a^2 \sin(2\Omega t) \quad \left(\text{GW frequency} = 2\Omega \right)$$

- And the **GW luminosity** of the system is (we use Kepler's 3rd law $\Omega^2 = GM/a^3$)

$$L_{\text{GW}} = -\frac{dE}{dt} = \frac{G}{5c^5} (\mu \Omega a^2)^2 \langle 2 \sin^2(2\Omega t) + 2 \cos^2(2\Omega t) \rangle$$

$$= \frac{32}{5} \frac{G}{c^5} \mu^2 a^4 \Omega^6 = \frac{32}{5} \frac{G^4}{c^5} \frac{M^3 \mu^2}{a^5}$$

GW emission from a binary system (III)

- The **total energy** of the binary system can be written as :

$$E = \frac{1}{2} \Omega^2 (M_1 a_1^2 + M_2 a_2^2) - \frac{GM_1 M_2}{a} = -\frac{1}{2} \frac{G\mu M}{a}$$

- As the gravitating system loses energy by emitting radiation, the distance between the two bodies **shrinks at a rate**:

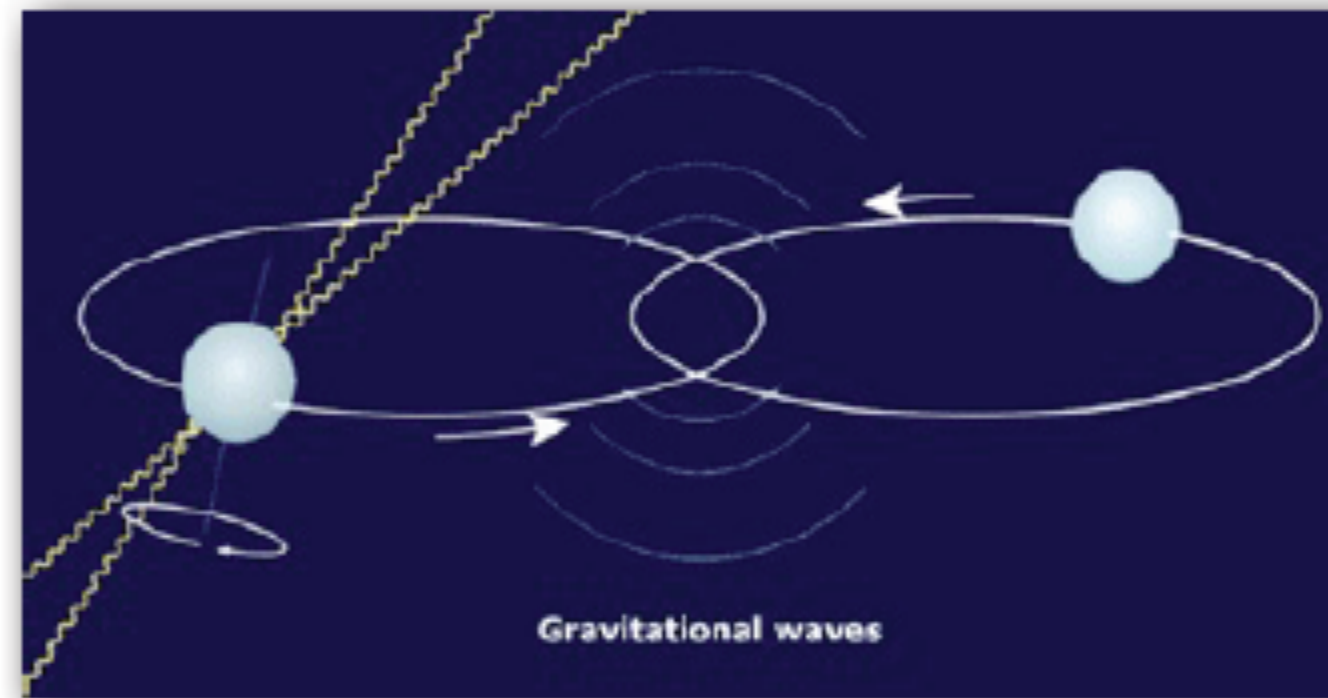
$$\frac{dE}{dt} = \frac{G\mu M}{2a^2} \frac{da}{dt} \rightarrow \frac{da}{dt} = -\frac{64}{5} \frac{G^3}{c^5} \frac{\mu M}{a^3}$$

- The orbital frequency increases accordingly $\dot{T}/T = (3/2)\dot{a}/a$.

- The system will **coalesce** after a time: $\tau = \frac{5}{256} \frac{c^5}{G^3} \frac{a_0^4}{\mu M^4}$ (initial separation)

PSR 1913+16: a Nobel-prize GW source

- The now famous [Hulse & Taylor](#) binary neutron star system provided the first astrophysical evidence of the existence of GWs !



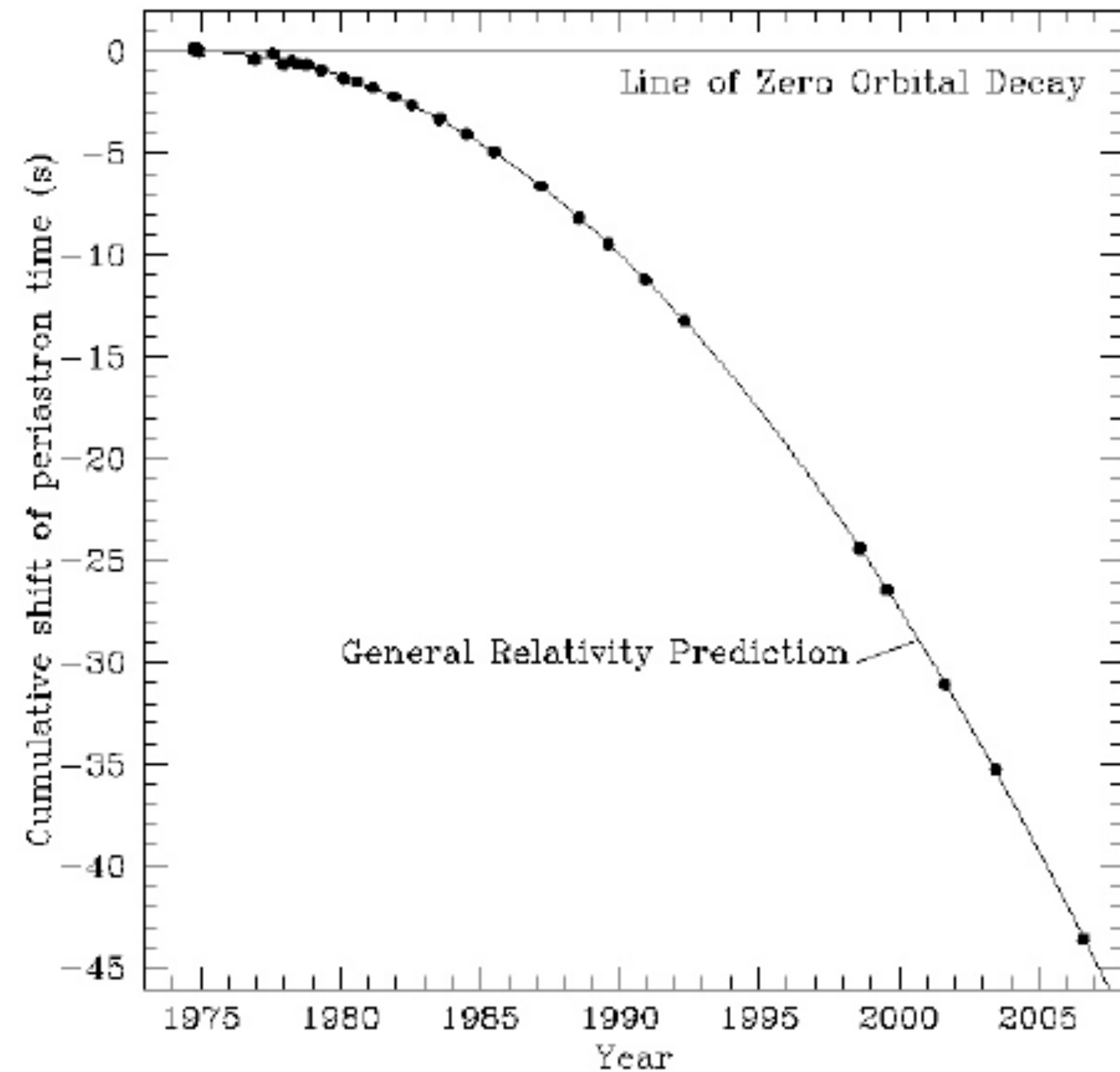
- The system's parameters: $r = 5 \text{ Kpc}$, $M_1 \approx M_2 \approx 1.4 M_\odot$, $T = 7 \text{ h } 45 \text{ min}$

- Using the previous equations we can predict:

$$\dot{T} = -2.4 \times 10^{-12} \text{ sec/sec}, \quad f_{\text{GW}} = 7 \times 10^{-5} \text{ Hz}, \quad h \sim 10^{-23}, \quad \tau \approx 3.5 \times 10^8 \text{ yr}$$

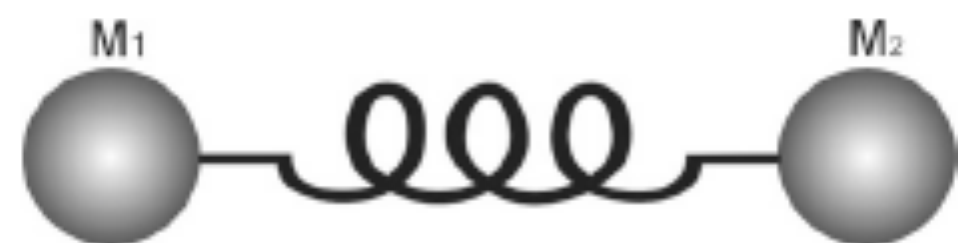
Theory vs observations

- How can the orbital parameters be measured with such high precision?
- One of the neutron stars is a **pulsar**, emitting extremely stable periodic radio pulses. The emission is modulated by the orbital motion.
- Since the discovery of the H-T system in 1974 more such binaries were found by astronomers.



A toy model GW detector

- Consider a GW propagating along the z-axis (with a “+” polarization and frequency ω), impinging on an idealized detector consisting of two masses joined by a spring (of length L) along the x-axis



- The resulting motion is that of a forced oscillator (with friction τ , natural frequency ω_0):

$$\ddot{\xi} + \dot{\xi}/\tau + \omega_0^2 \xi = -\frac{1}{2}\omega^2 L h_+ e^{i\omega t}$$

- The solution is:

$$\xi = \frac{\omega^2 L h_+}{2(\omega_0^2 - \omega^2 + i\omega/\tau)} e^{i\omega t}$$

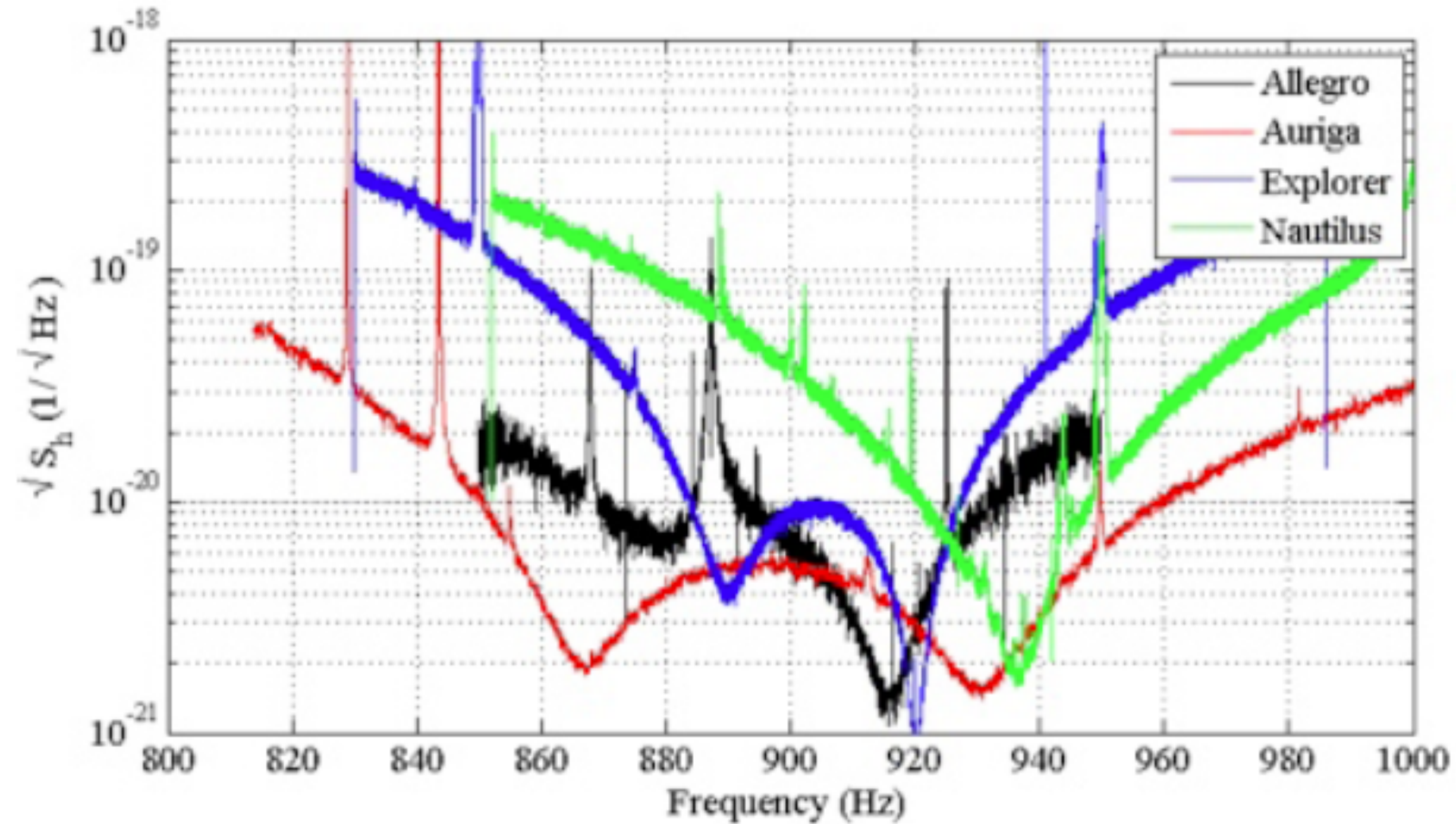
- The **maximum amplitude** is achieved at $\omega \approx \omega_0$ and has a size: $\xi_{\max} = \frac{1}{2}\omega_0 \tau L h_+$

- The detector can be optimized by increasing $\omega_0 \tau L$.

Bar detectors

J. Weber

- Bar detectors are narrow bandwidth instruments (like the previous toy-model)

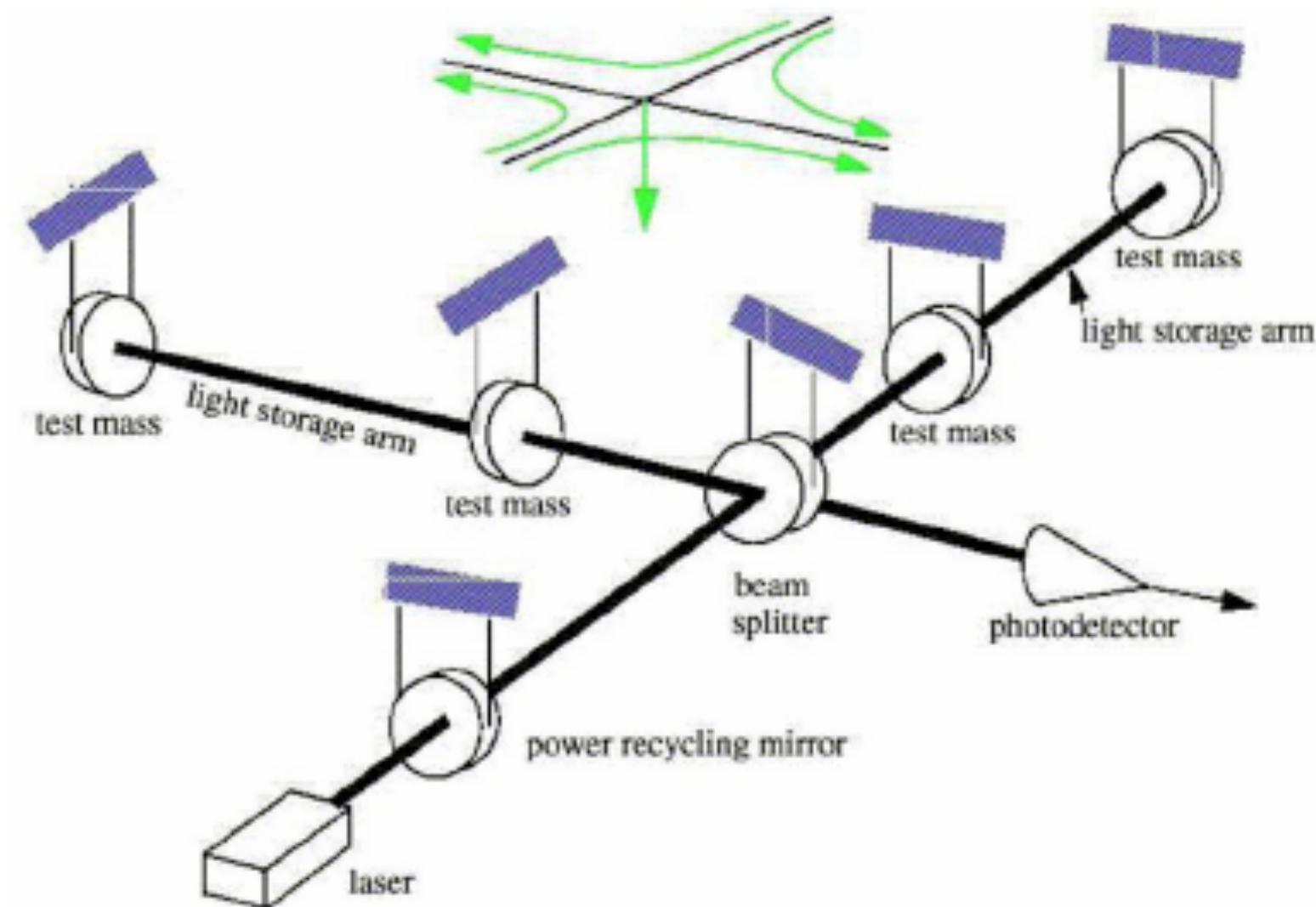


Sensitivity curves of various bar detectors



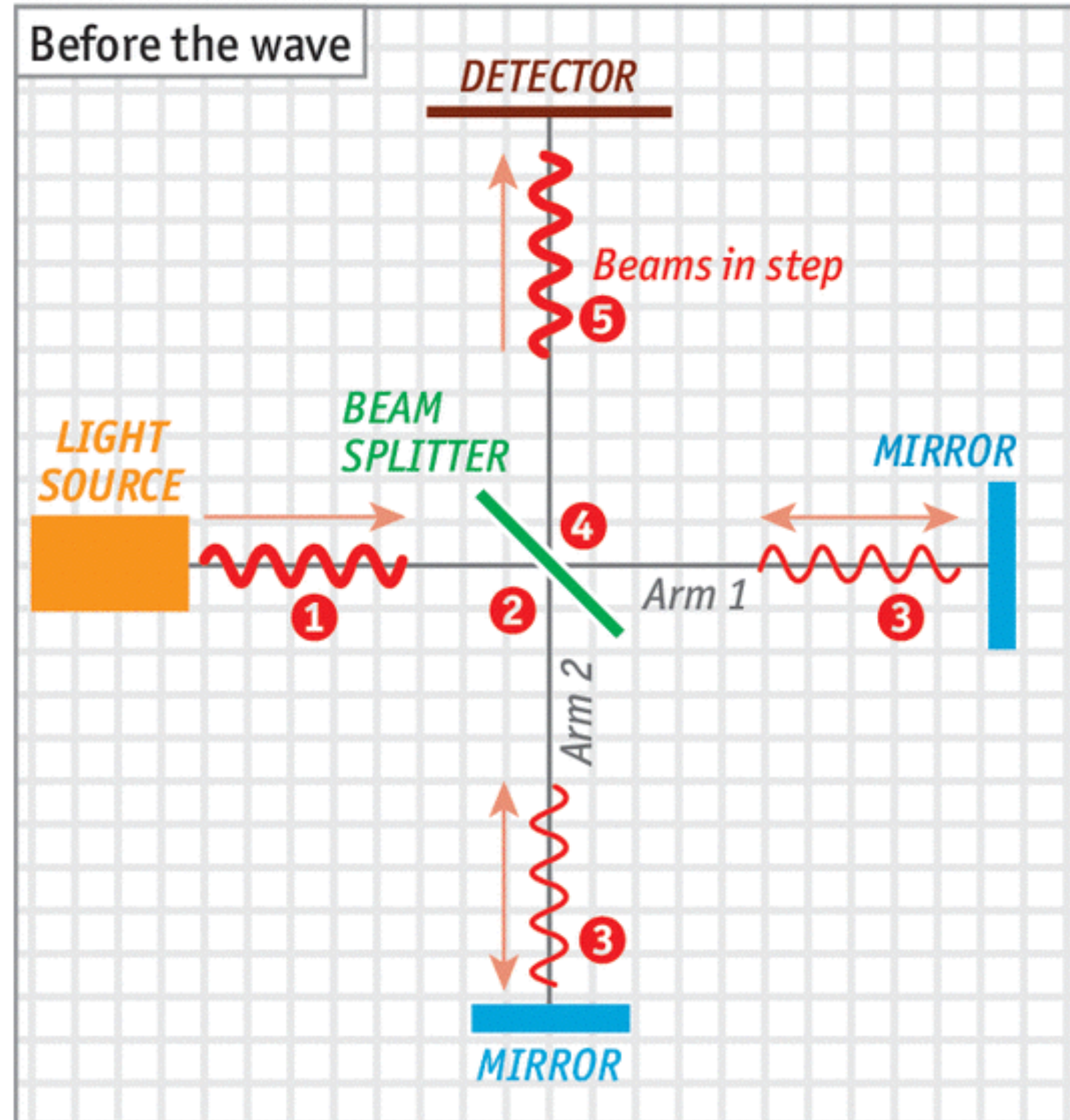
Detectors: laser interferometry

- A laser interferometer is an alternative choice for GW detection, offering a combination of **very high sensitivities over a broad frequency band**.
- **Suspended mirrors** play the role of “test-particles”, placed in perpendicular directions. The light is reflected on the mirrors and returns back to the beam splitter and then to a photodetector where the fringe pattern is monitored.

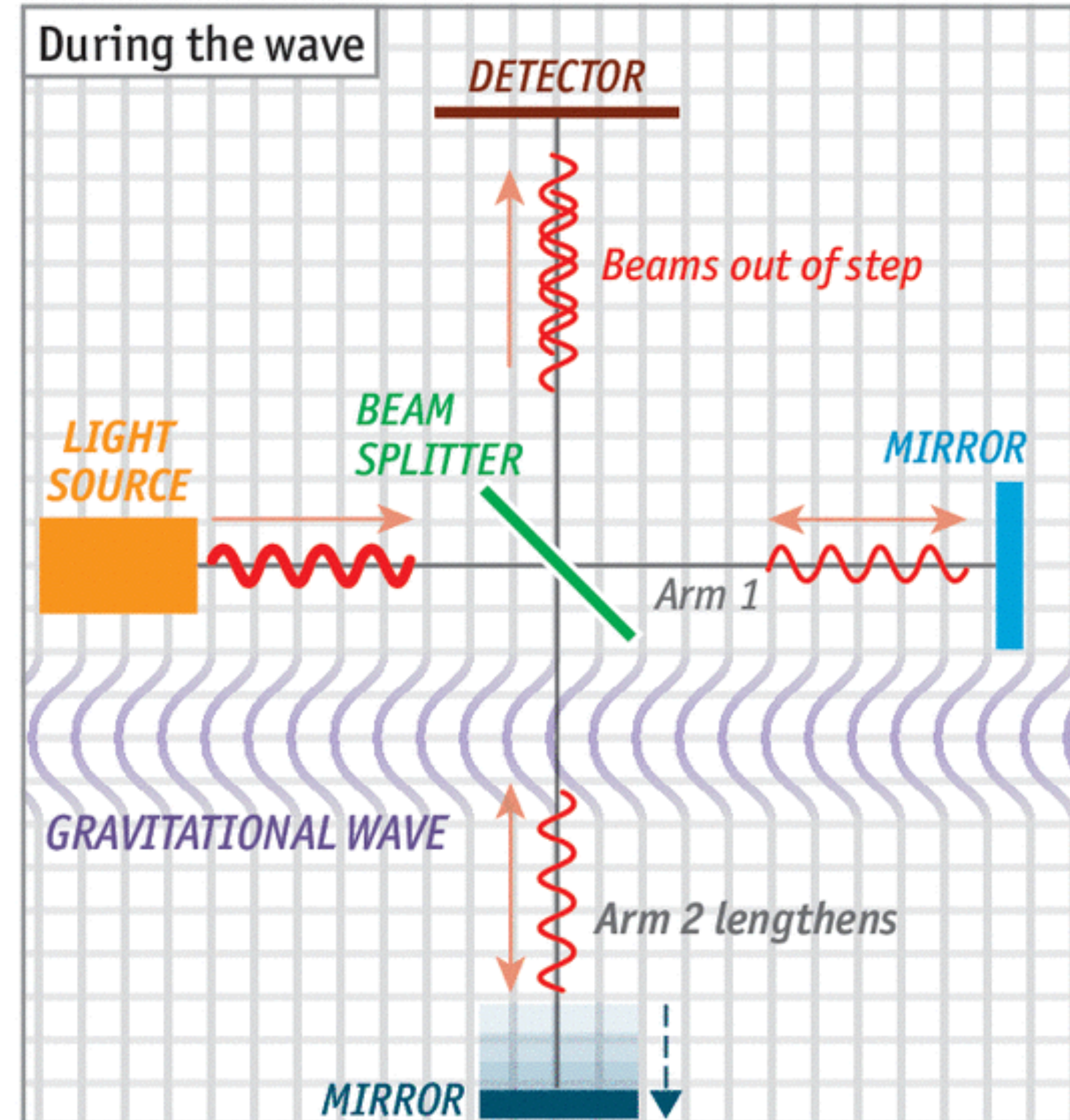


Catching a wave

How a laser-interferometer observatory works



The **light source** sends out a **beam** ① that is divided by a **beam splitter** ②. The half-beams produced follow paths of identical length ③, reflecting off **mirrors** to recombine ④, then travel in step to the **detector** ⑤.



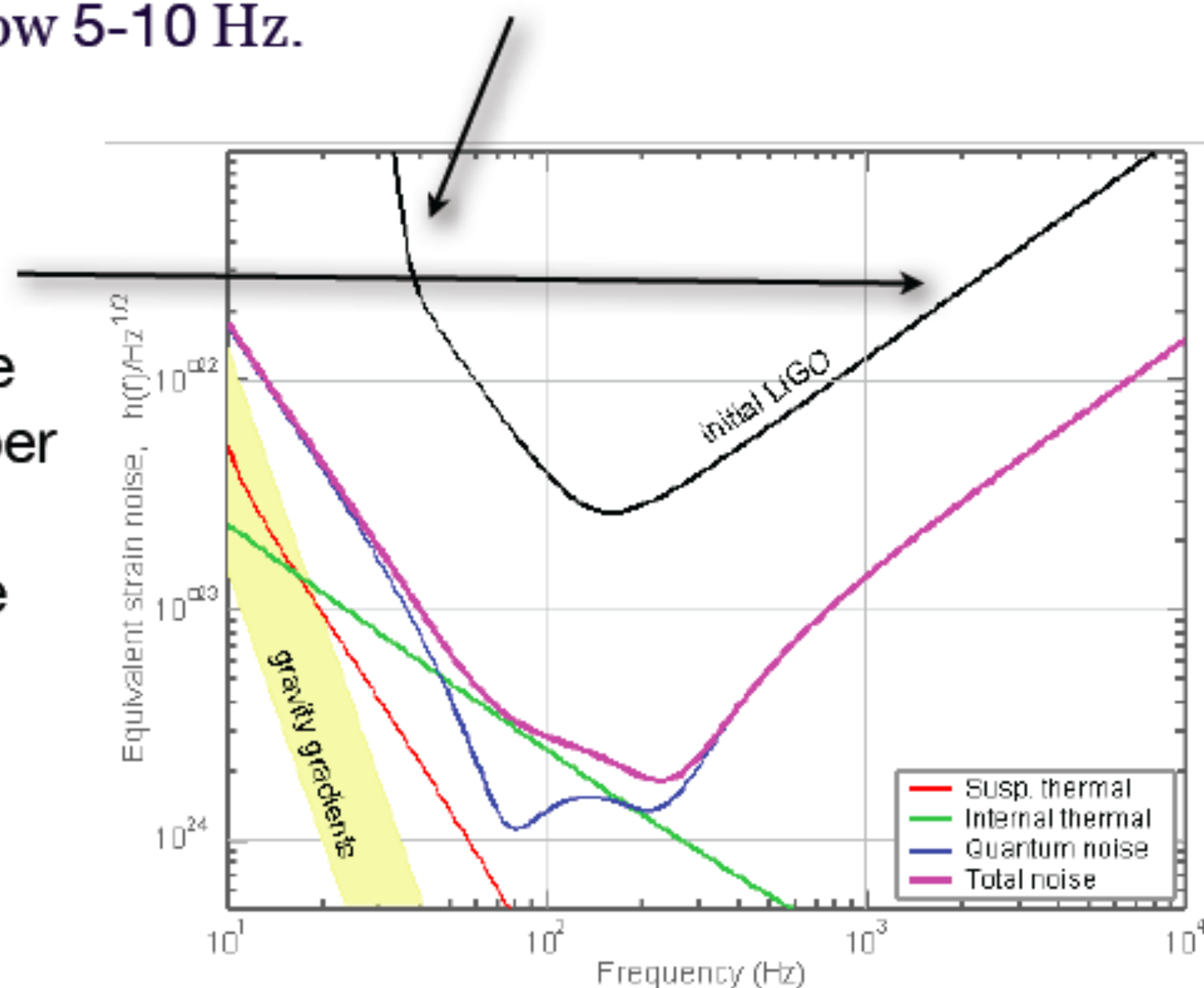
When a **gravitational wave** arrives, it disturbs space-time, lengthening (in this example) the light's path along **arm 2**; when the **beams** recombine and arrive at the **detector**, they are no longer in step.

Source: *The Economist*

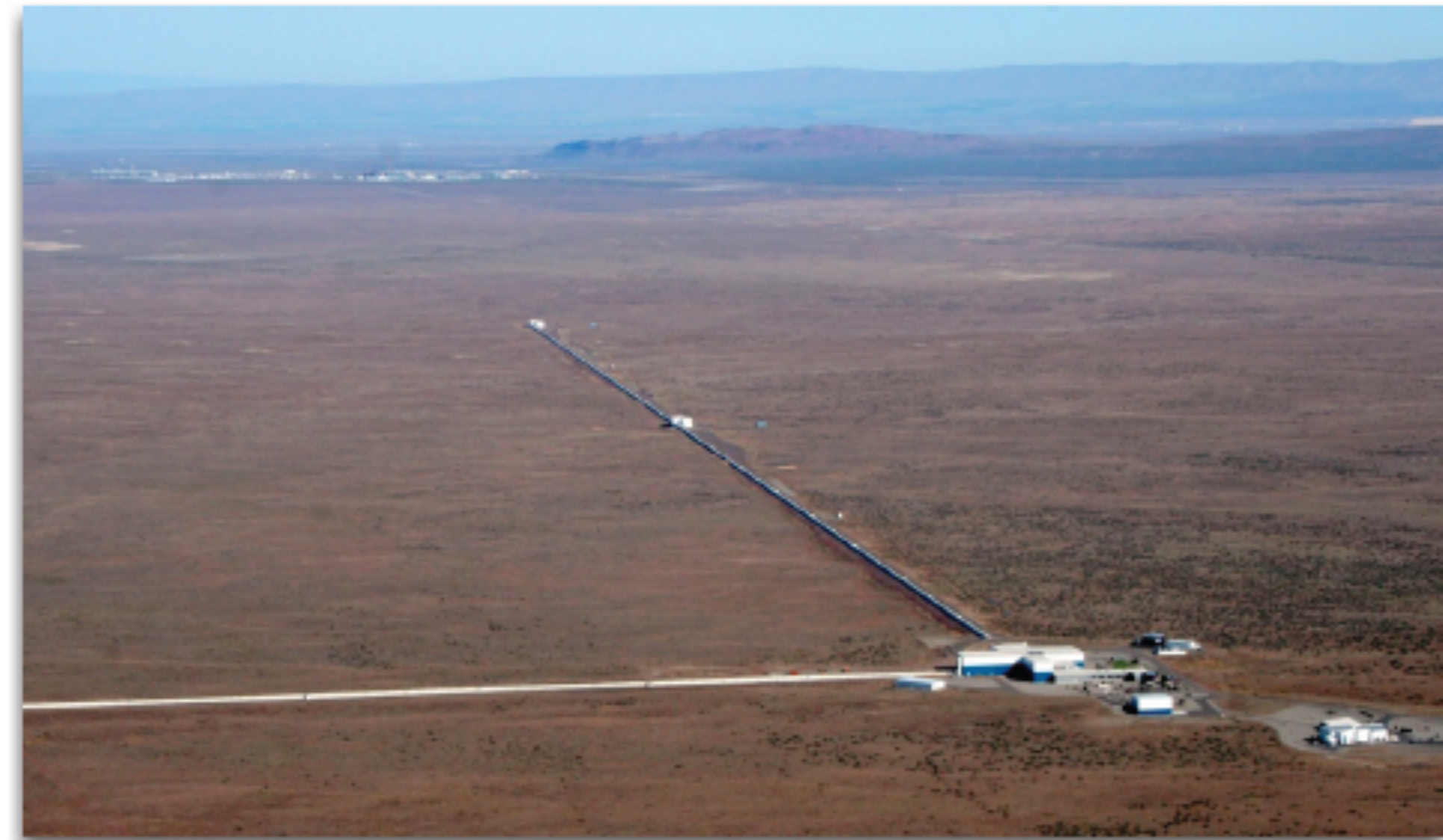
Noise in interferometric detectors

- **Seismic noise (low frequencies).** At frequencies below 60 Hz, the noise in the interferometers is dominated by seismic noise. The vibrations of the ground couple to the mirrors via the wire suspensions which support them. This effect is strongly suppressed by properly designed suspension systems. Still, seismic noise is very difficult to eliminate at frequencies below 5-10 Hz.

- **Photon shot noise (high frequencies).** The precision of the measurements is restricted by fluctuations in the fringe pattern due to fluctuations in the number of detected photons. The number of detected photons is proportional to the intensity of the laser beam. Statistical fluctuations in the number of detected photons imply an uncertainty in the measurement of the arm length.

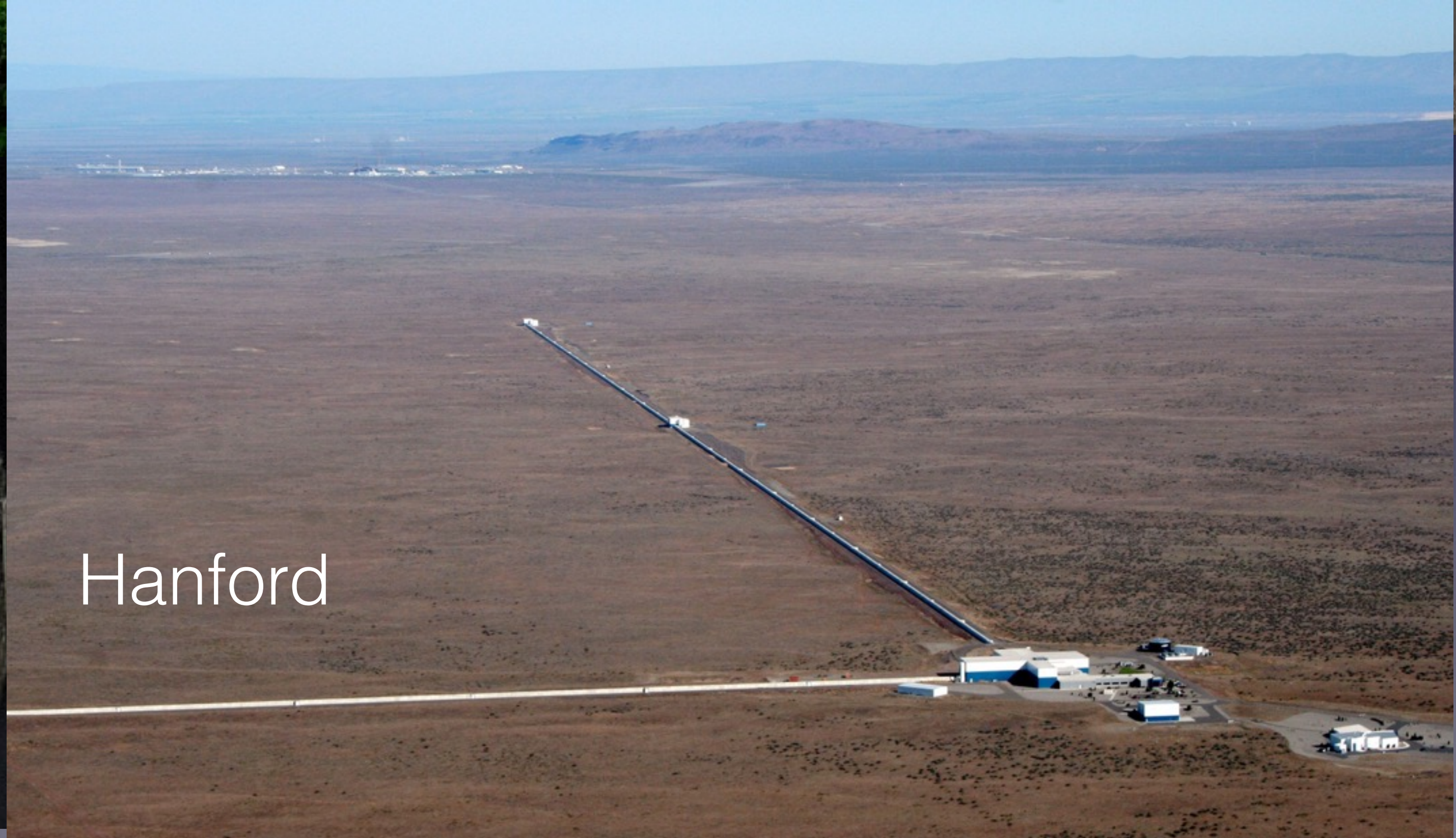


Detectors: the present (I)



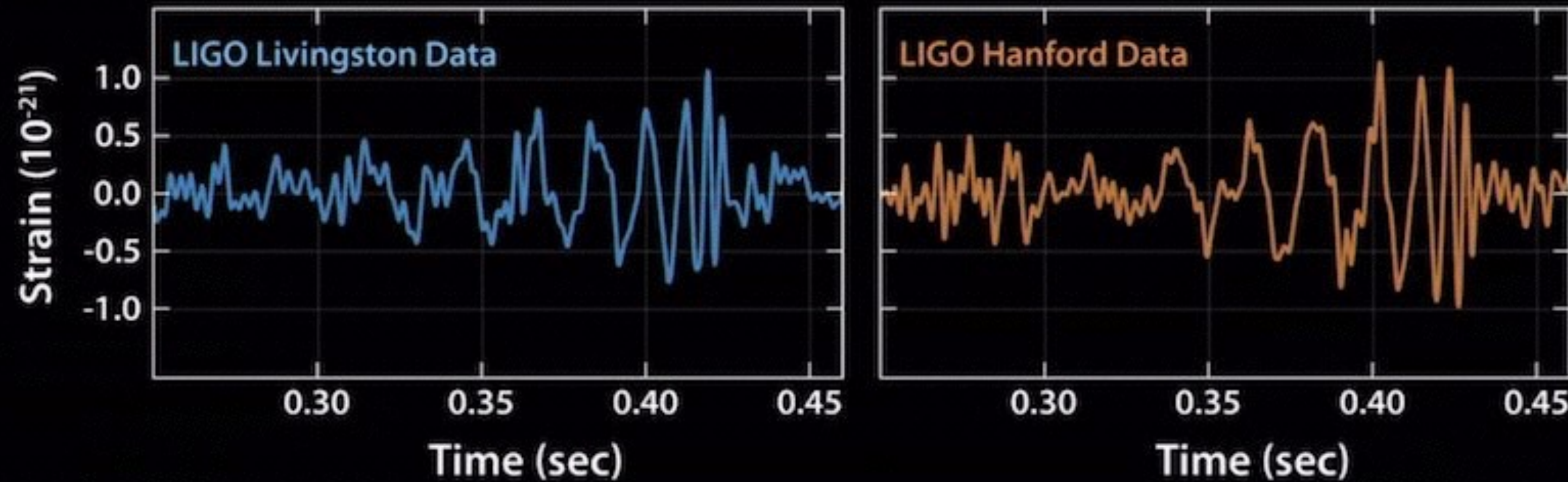
The twin LIGO detectors ($L = 4$ km) at Livingston Louisiana and Hanford Washington (US).

Livingston



Hanford

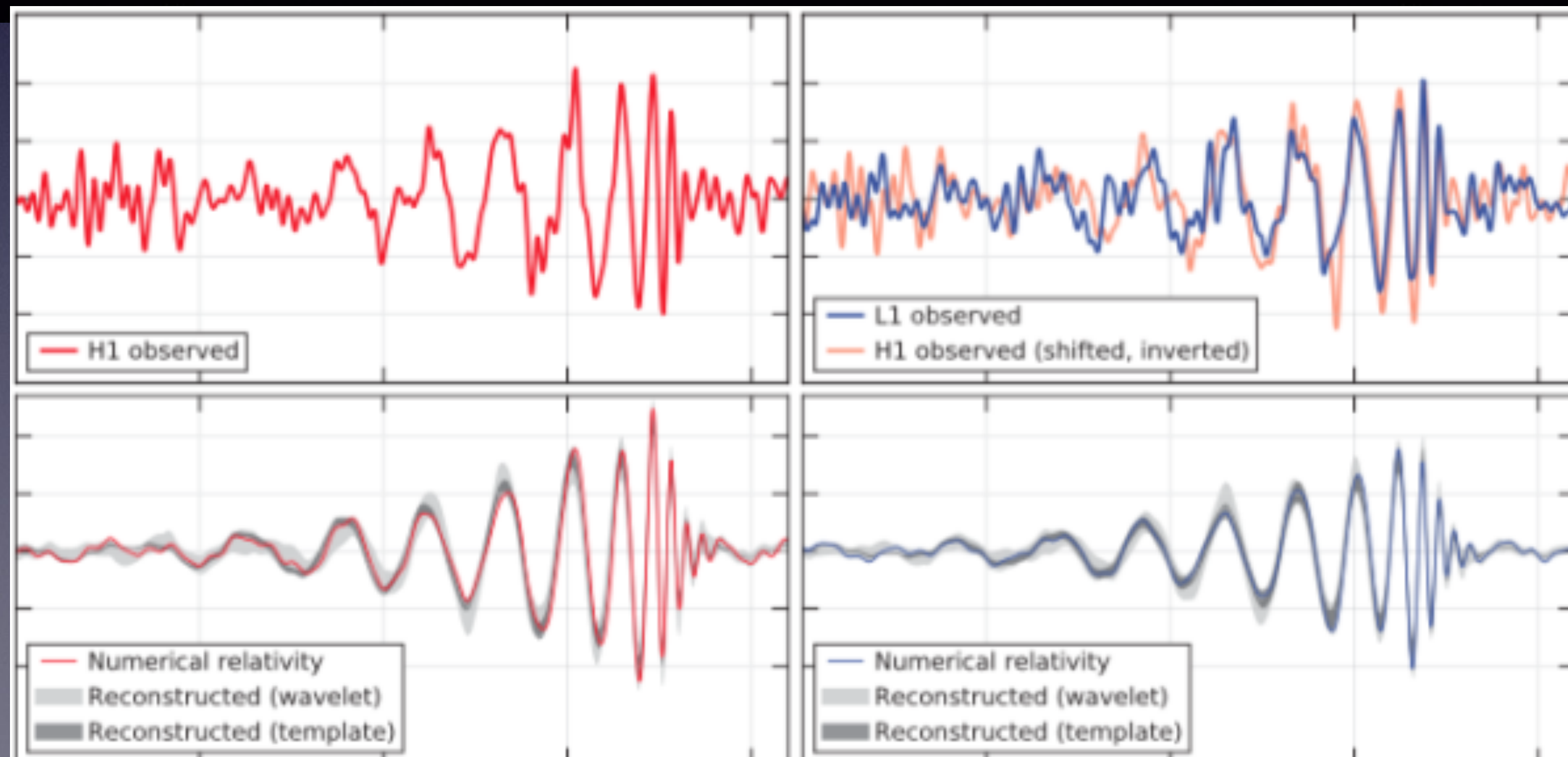
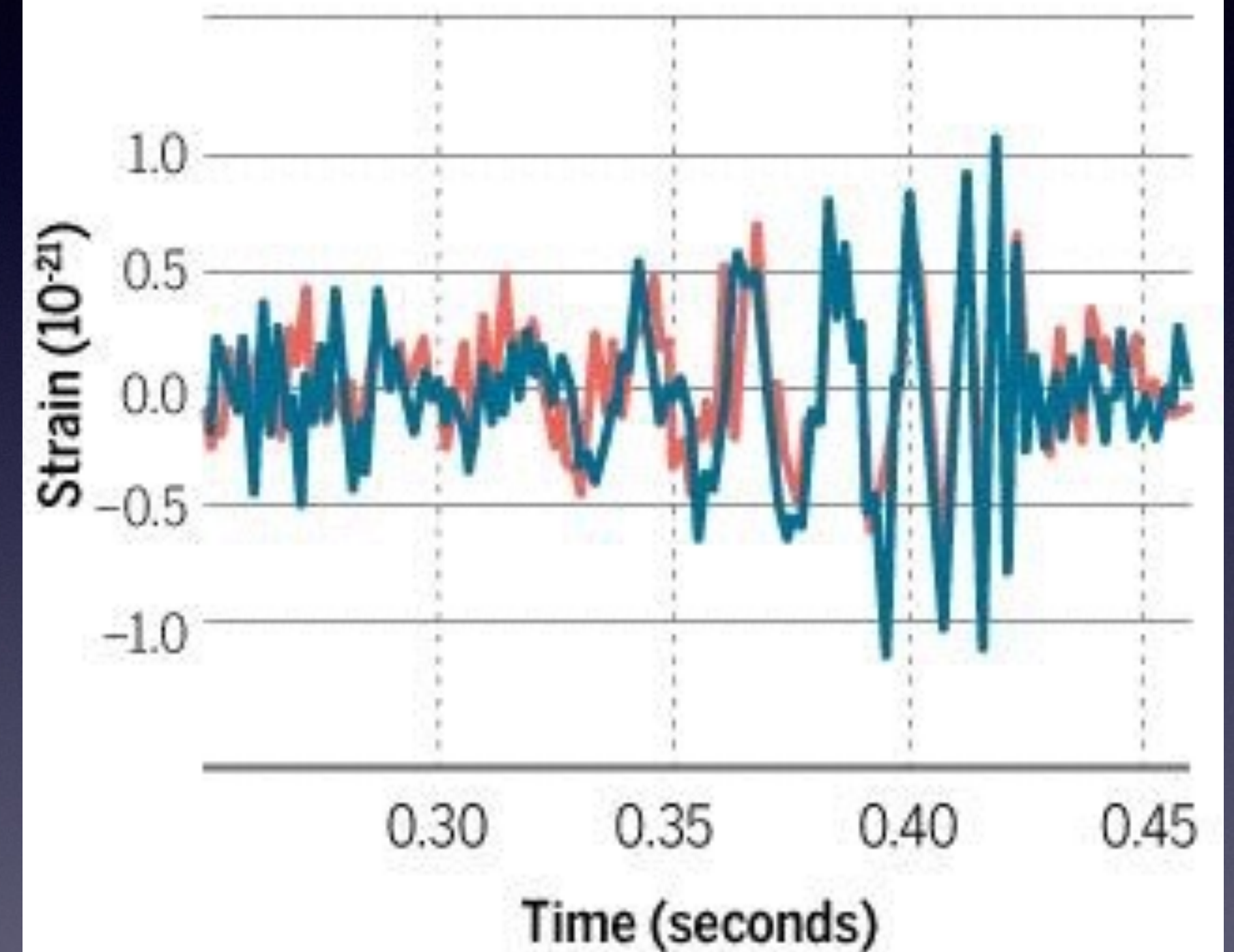
Gravitational waves detected by LIGO!



Signals in synchrony

When shifted by 0.007 seconds, the signal from LIGO's observatory in Washington (red) neatly matches the signal from the one in Louisiana (blue).

● LIGO Hanford data (shifted) ● LIGO Livingston data

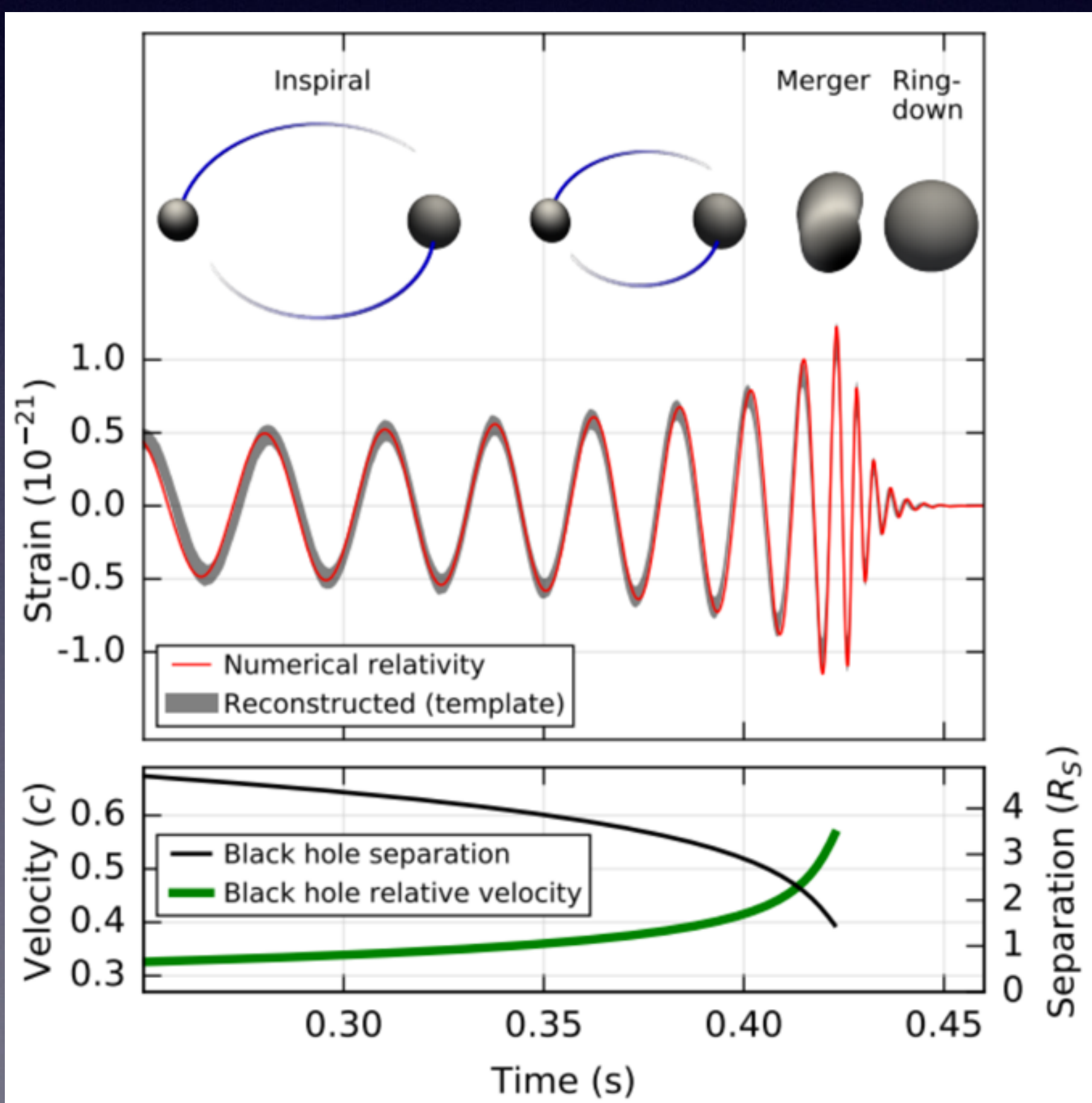


September 14th, 2015,
09:50:45 UTC.

Range: from 35 to 250 Hz

LIGO

The First Observation
of Gravitational Waves



Primary black hole mass

$$36_{-4}^{+5} M_{\odot}$$

Secondary black hole mass

$$29_{-4}^{+4} M_{\odot}$$

Final black hole mass

$$62_{-4}^{+4} M_{\odot}$$

Final black hole spin

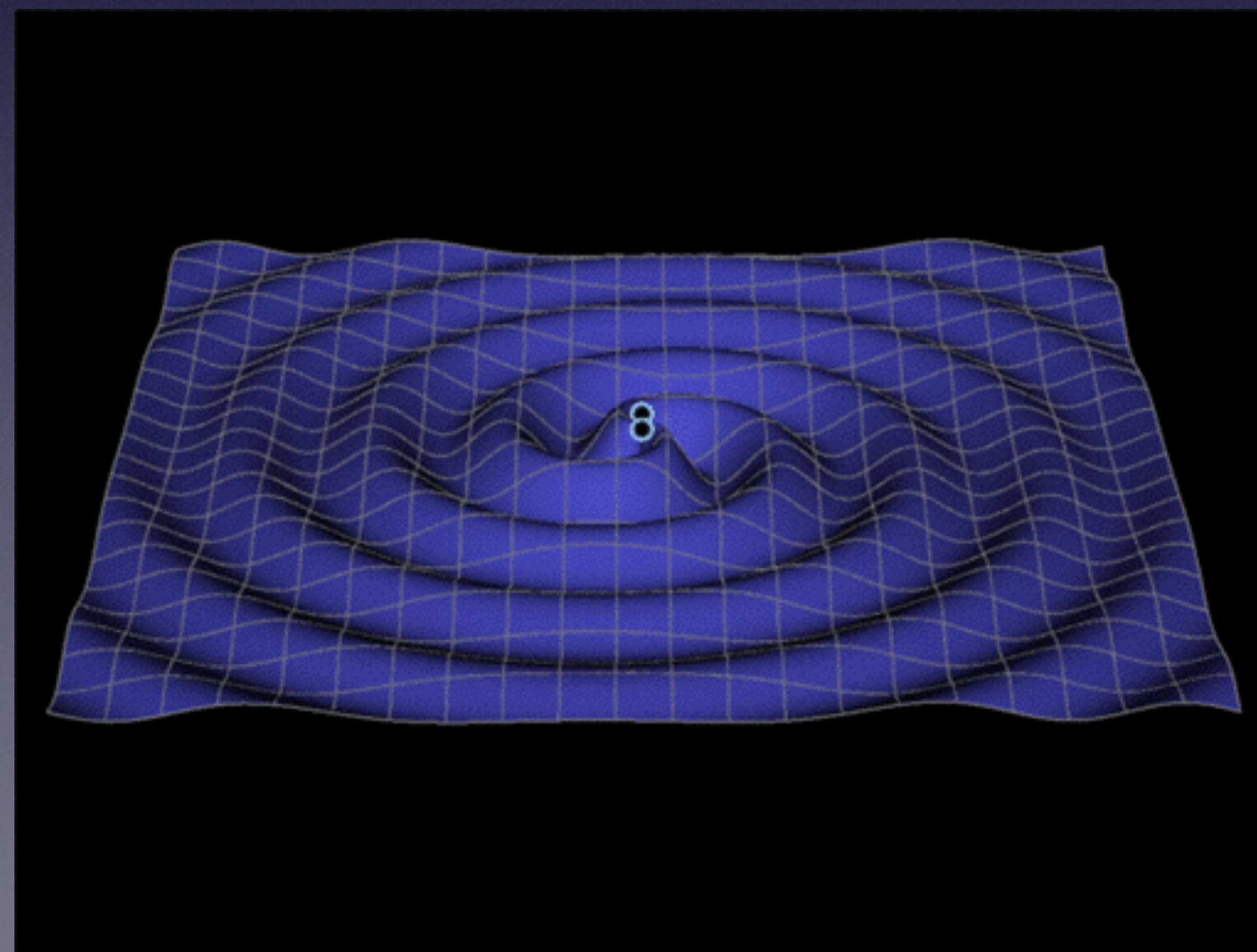
$$0.67_{-0.07}^{+0.05}$$

Luminosity distance

$$410_{-180}^{+160} \text{ Mpc}$$

Source redshift z

$$0.09_{-0.04}^{+0.03}$$

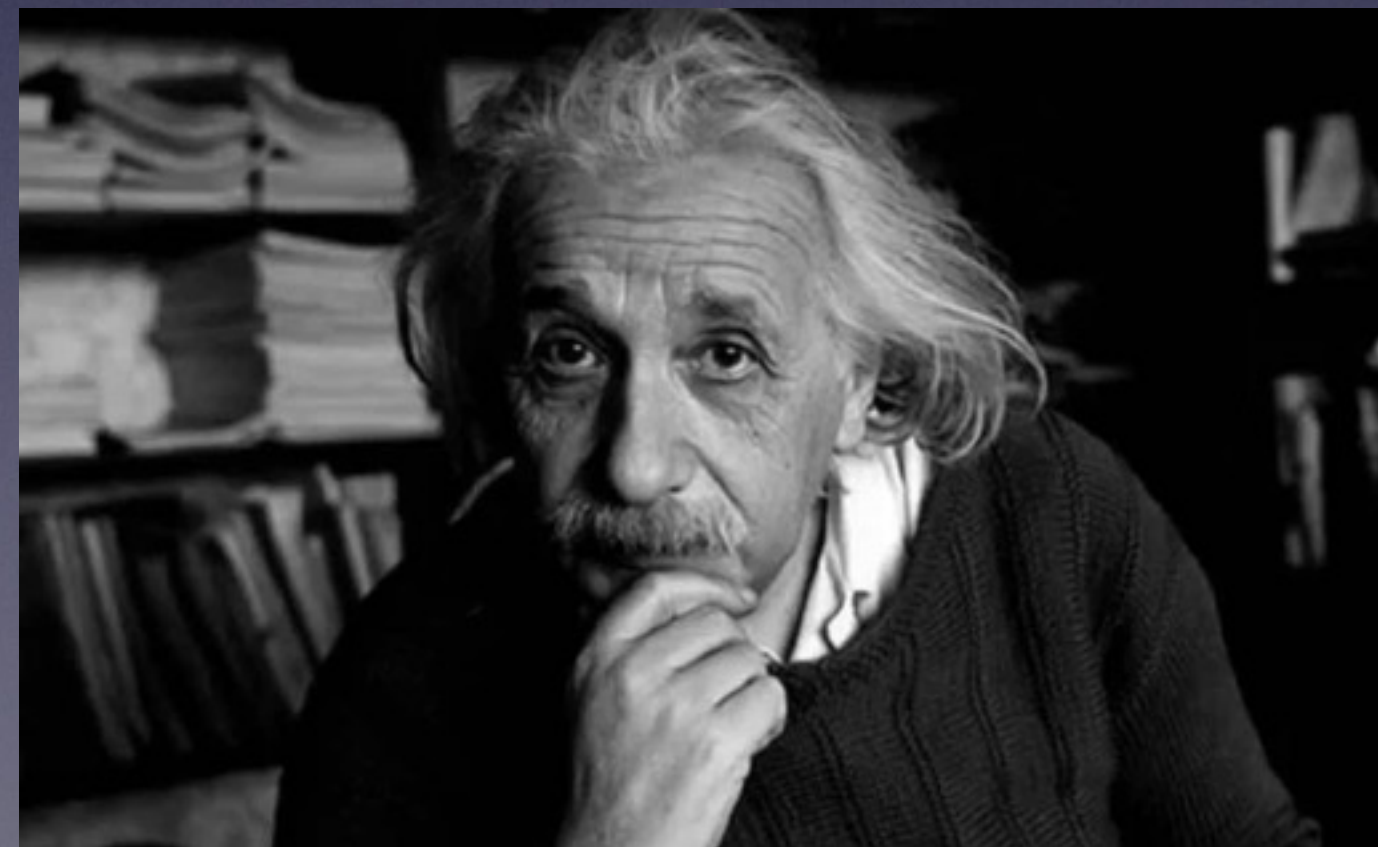


Implications of the detection:

- Gravitational waves exist
- Compact objects very much like to black holes exist
- Gravitational waves transport energy —> the gravitational field has energy in absence of matter/radiation
- Spacetime has a dimensionality of $n=4$ or higher.
- Existence is non-local.

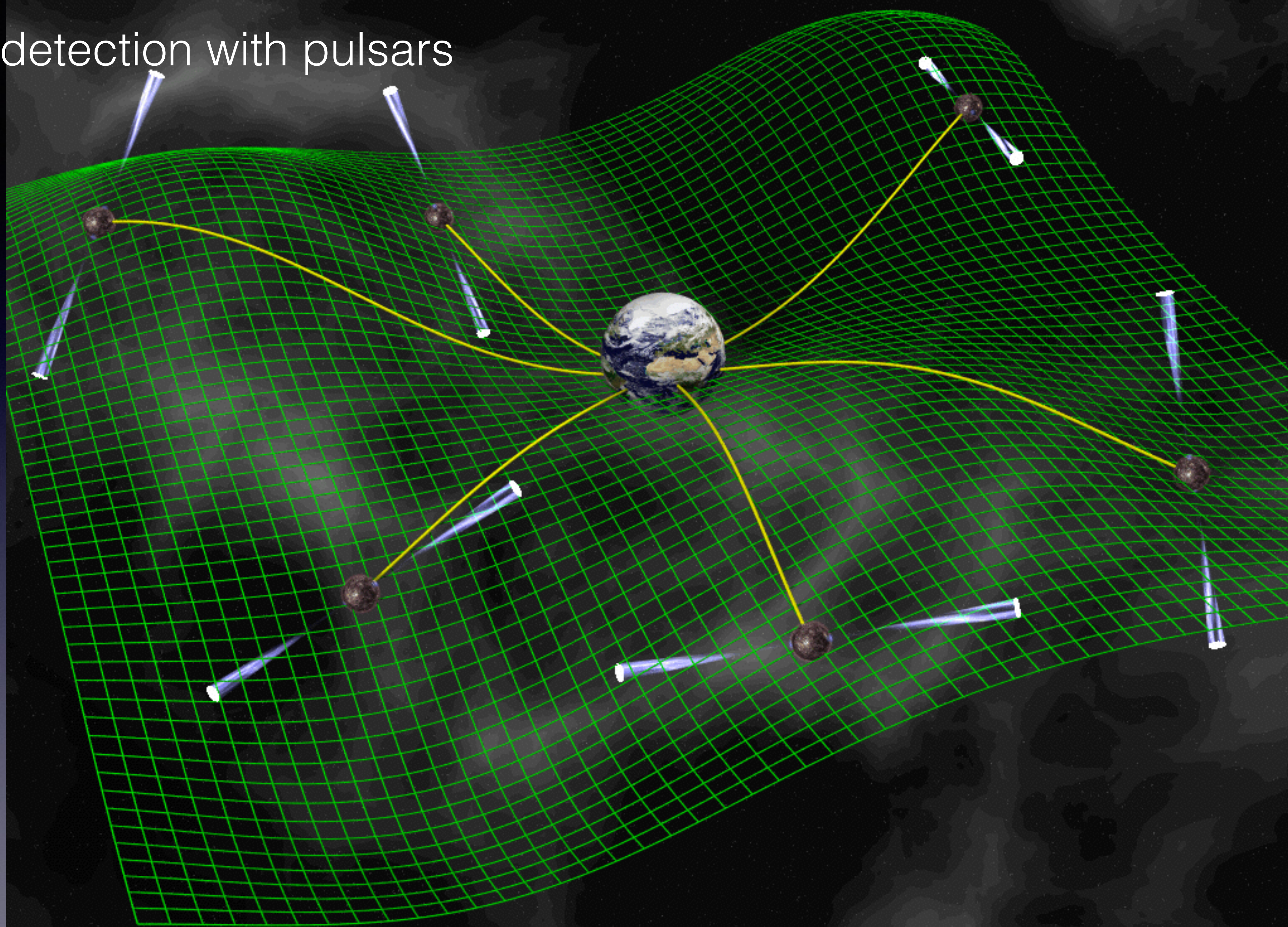
Gravitational wave astronomy is born!

“He’s looking at you kiddo”



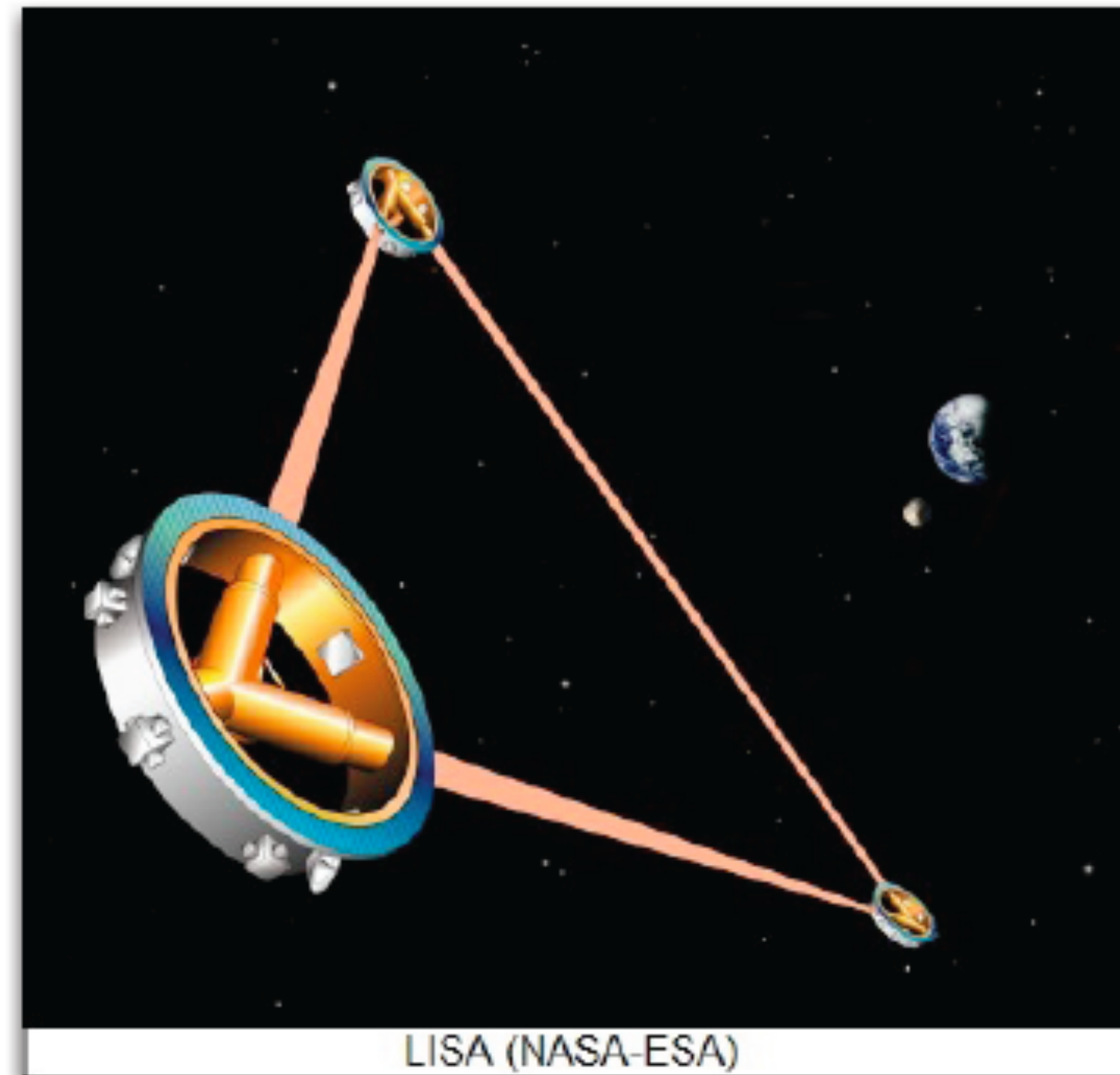
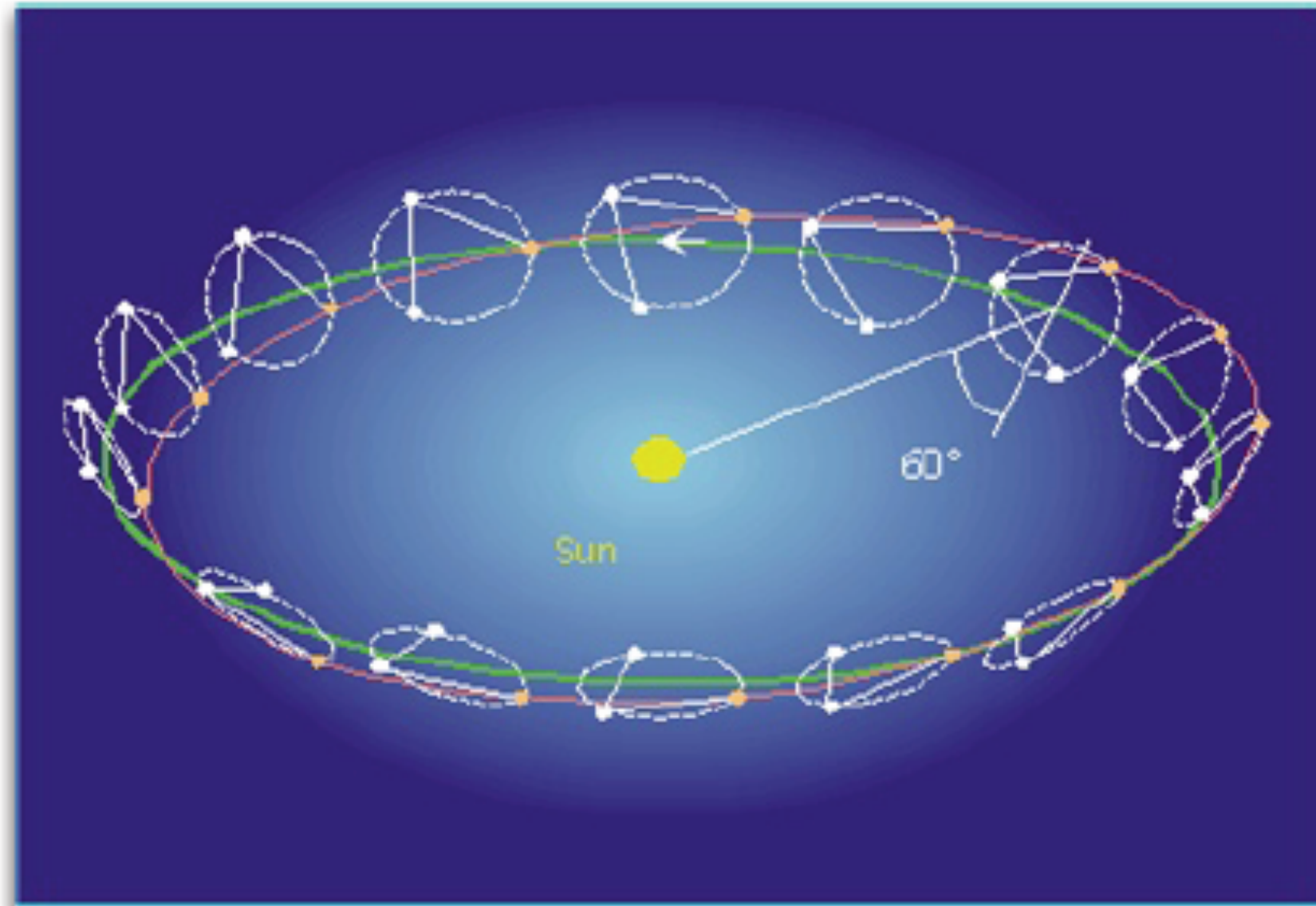
Gravitational wave detection with pulsars

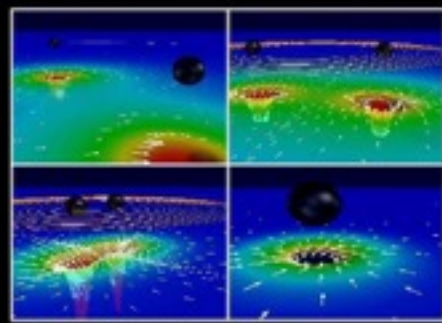
EPTA/LEAP
IPTA



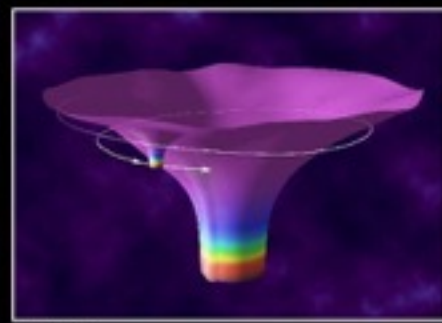
Going to space: the LISA detector

- Space-based detectors: “noise-free” environment, abundance of space!
- Long-arm baseline, **low frequency sensitivity**
- **LISA**: Up until recently a joint NASA/ESA mission, now an ESA mission only. To be launched around 2020.





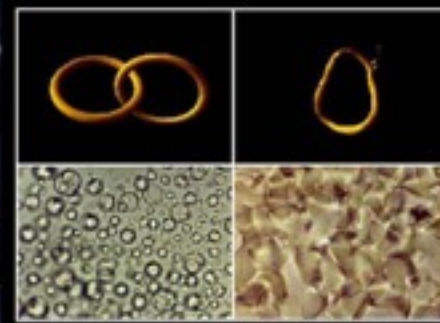
Supermassive Black Hole Binaries



Compact Object Captures



Galactic White Dwarf Binaries



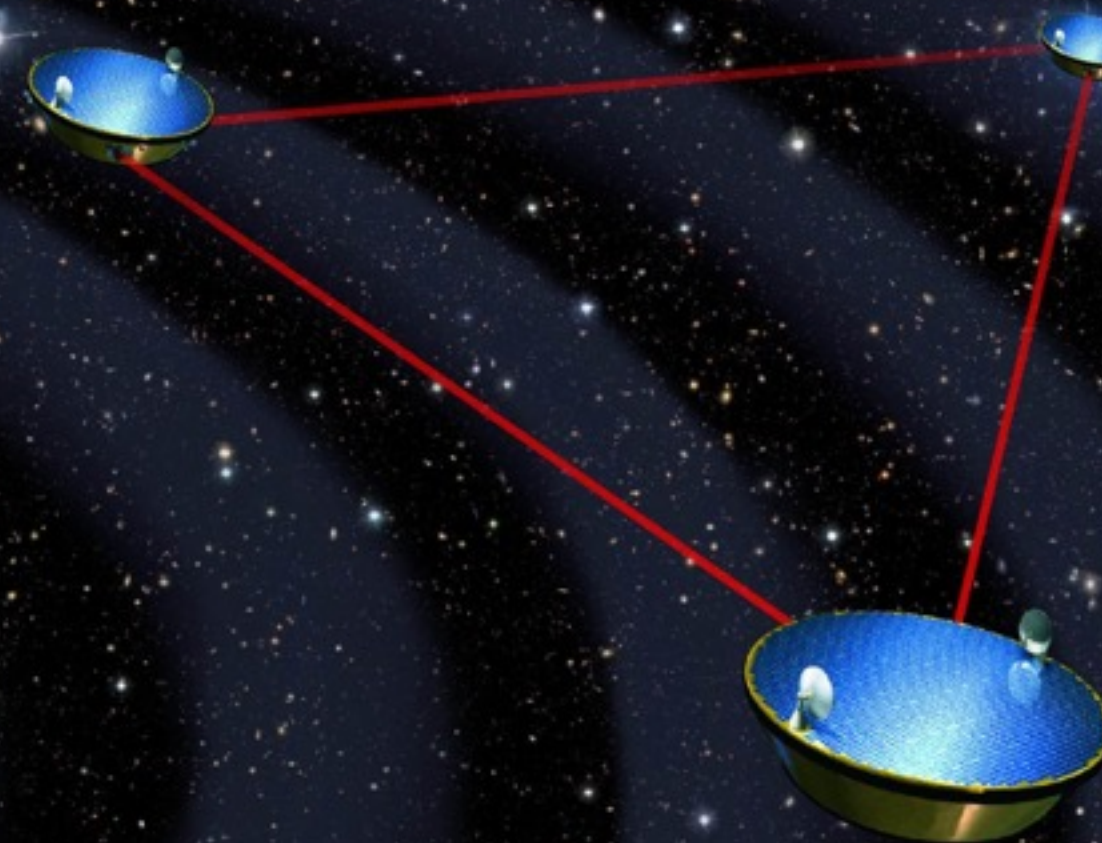
Cosmic Strings and Phase Transitions

LISA

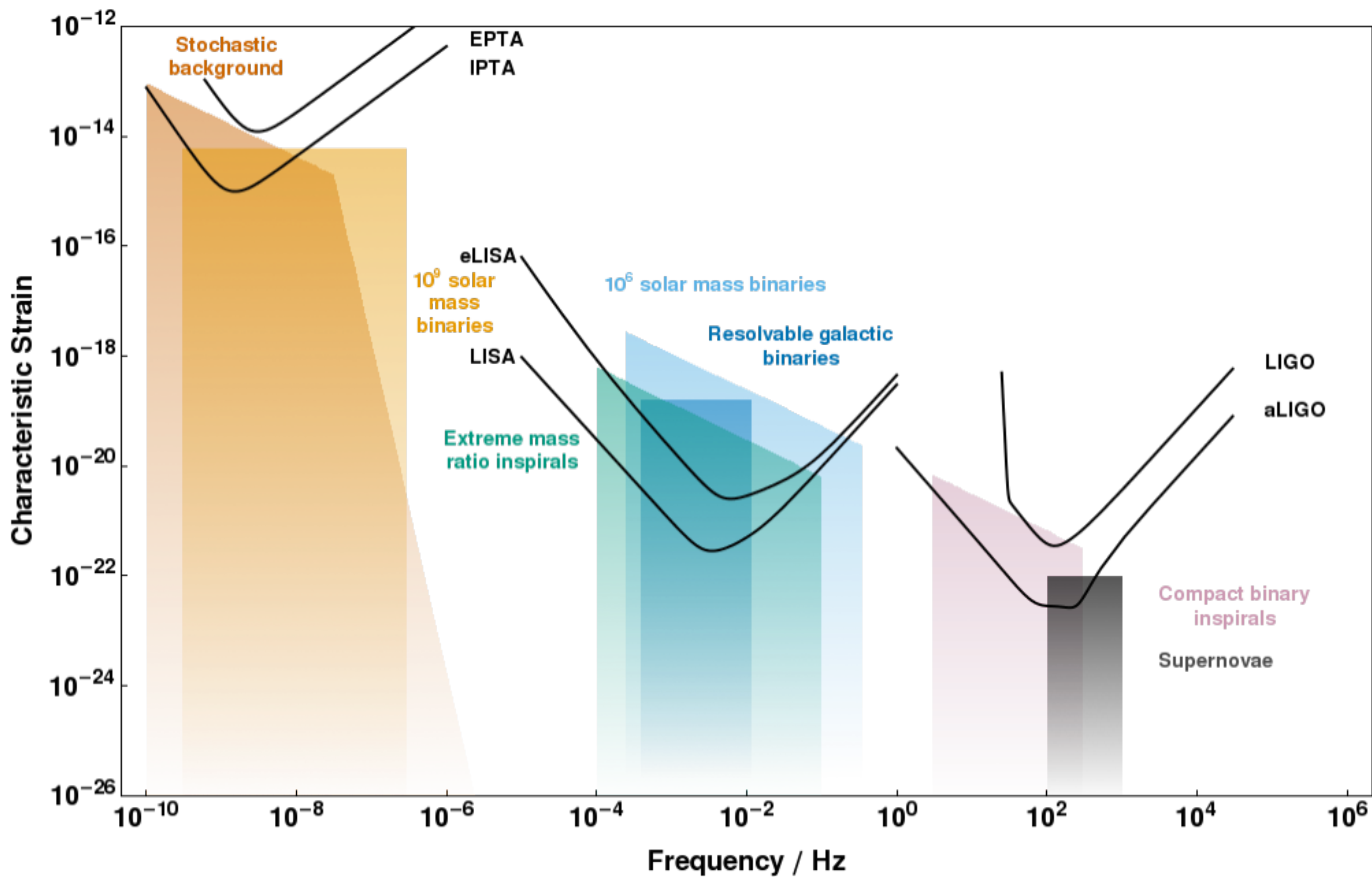
Laser Interferometer Space Antenna



Gravity is talking. LISA will listen.

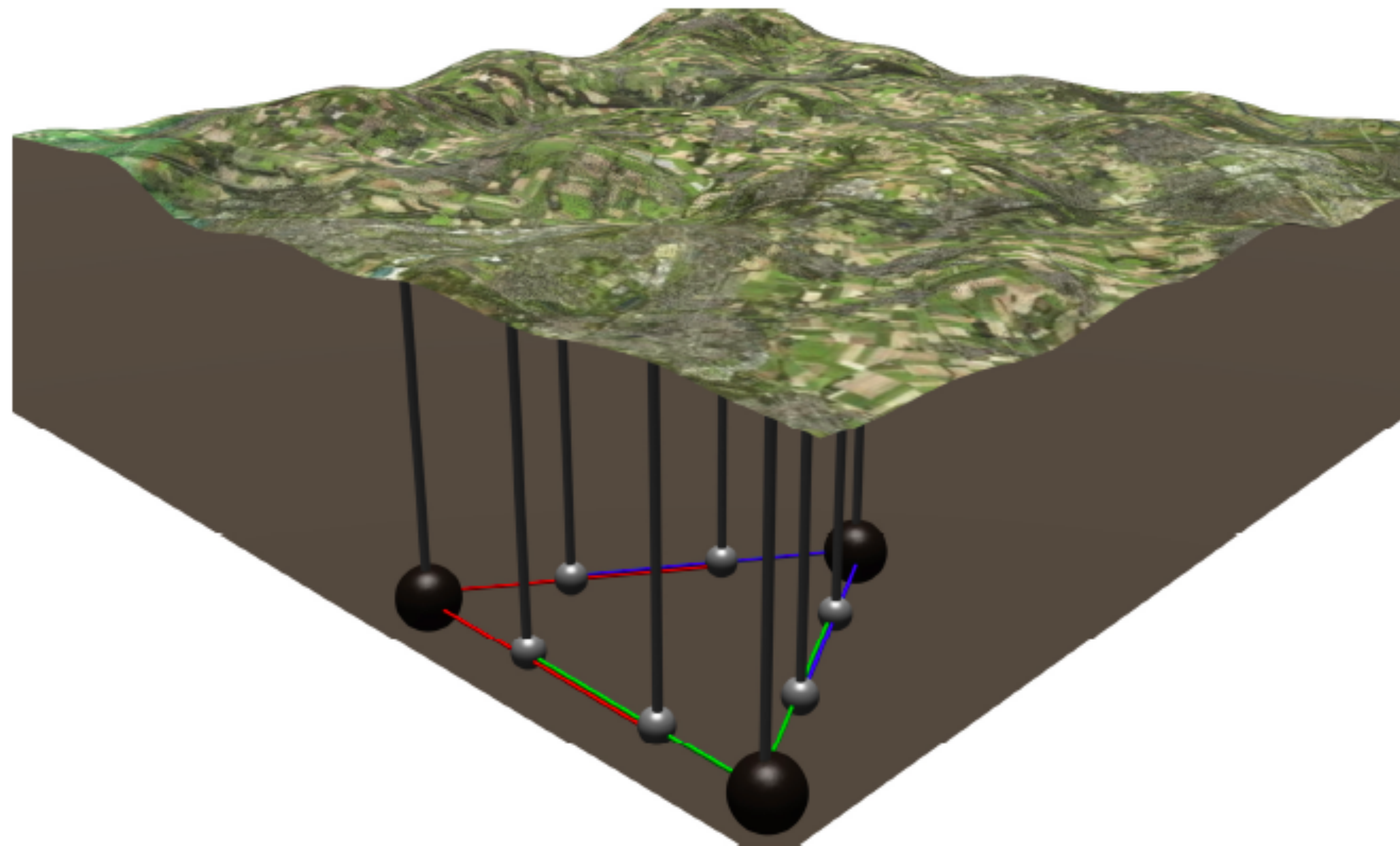


Black hole binary at $z=15$, $10^6 M_{\odot}$, two hours before merger. Numerical waveform plus instrument noise and WD background (J. Baker)



Going underground: the ET

- The [Einstein Telescope](#) will be the next generation underground detector.



The **Einstein Telescope** has been proposed by 8 European research institutes:

European Gravitational Observatory
Istituto Nazionale di Fisica Nucleare
Max Planck Society
Centre National de la Recherche Scientifique
University of Birmingham
University of Glasgow
NIKHEF
Cardiff University

The arms will be 10 km long (compared to 4 km for LIGO, and 3 km for Virgo), and like LISA, there will be three arms in an equilateral triangle, with two detectors in each corner.

The low-frequency interferometers (1 to 250 Hz) will use optics cooled to 10 K (-441.7 °F; -263.1 °C), with a beam power of about 18 kW in each arm cavity. The high-frequency ones (10 Hz to 10 kHz) will use room-temperature optics and a much higher recirculating beam power of 3 MW.

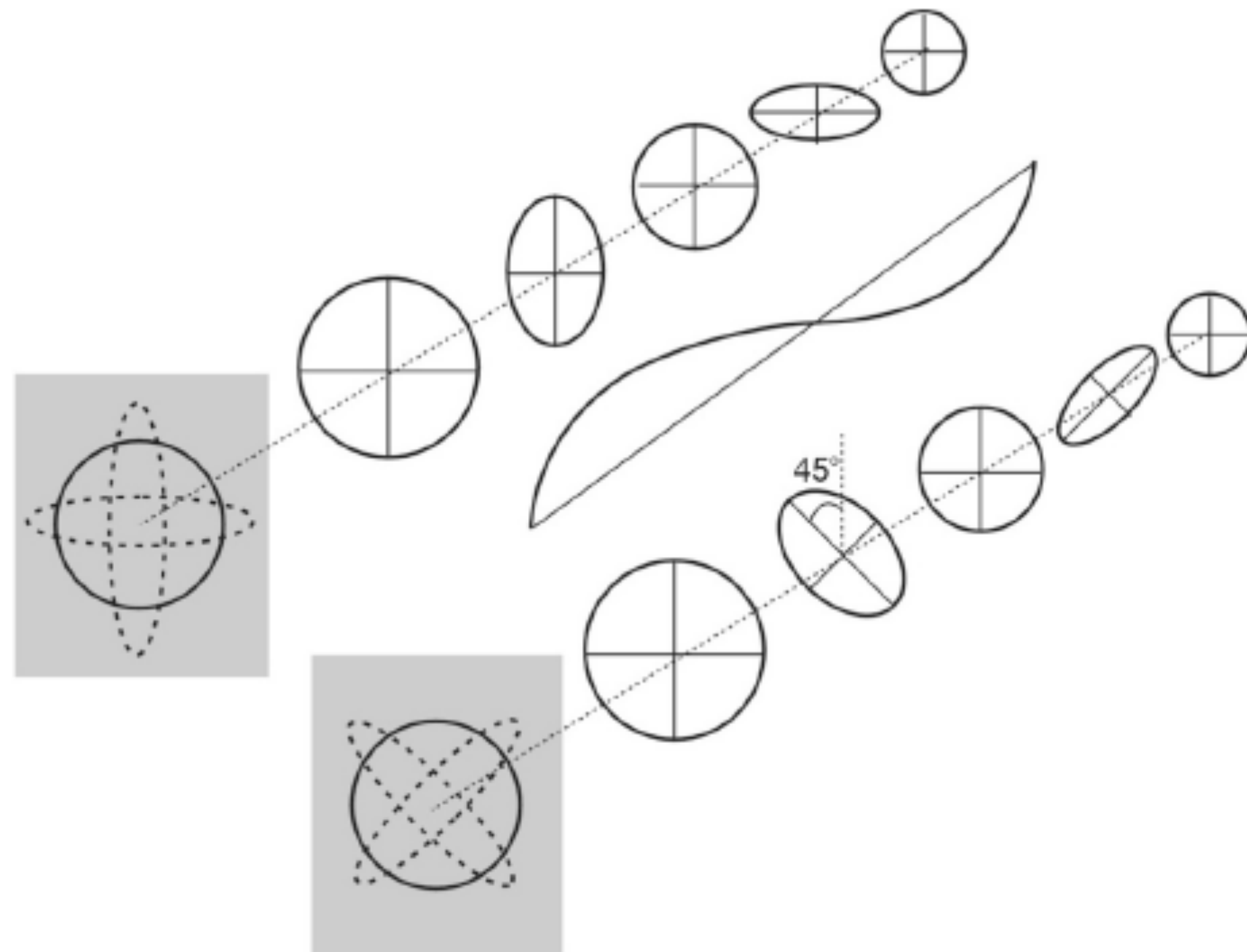


Thanks!

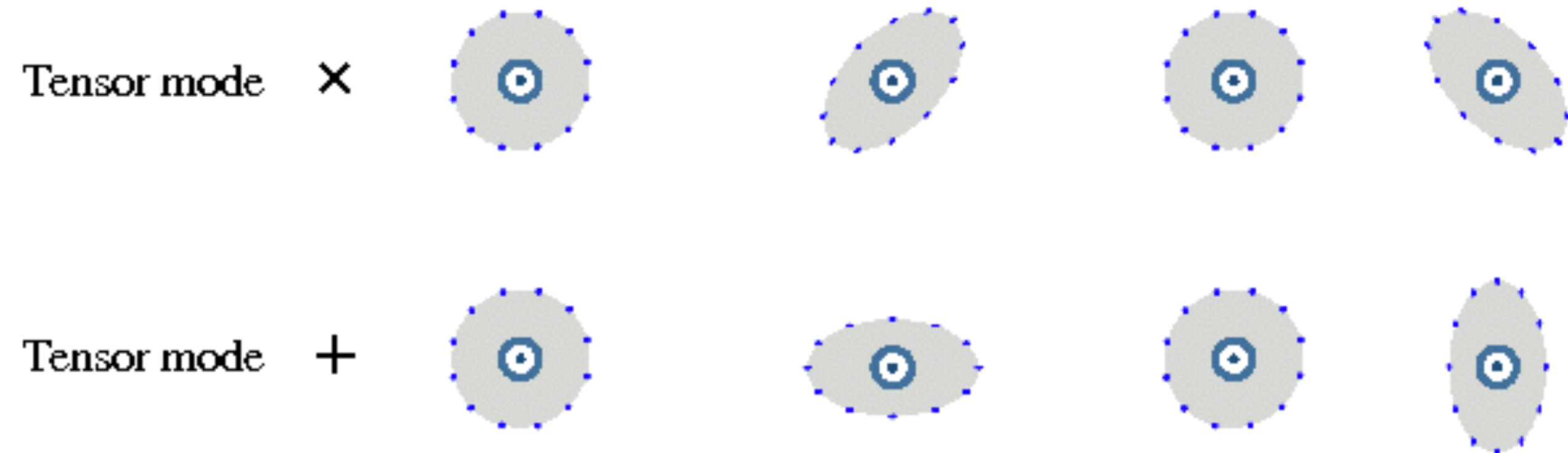
Effect on test particles (II)

- Similarly for a pair of particles placed on the y-axis:

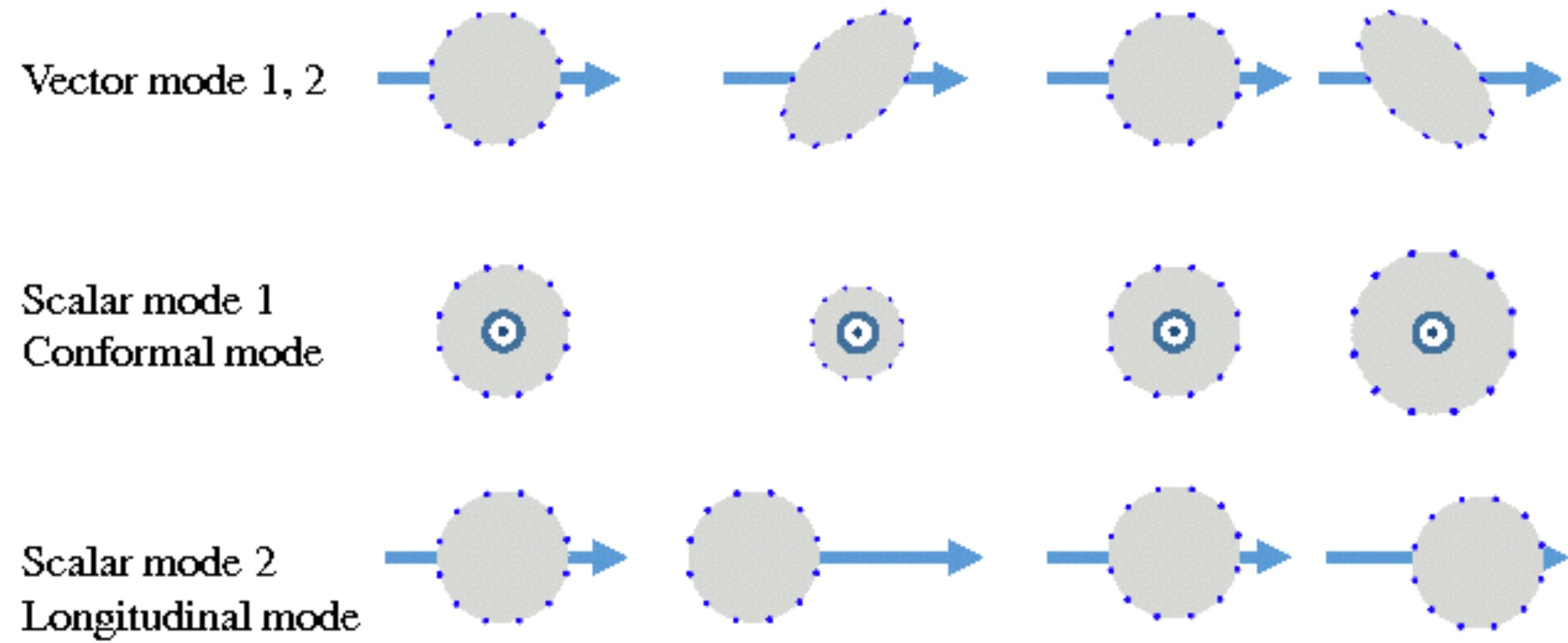
- Comment: the same result can be derived using the geodesic deviation equation. $dl \approx \left[1 + \frac{1}{2} h_+ \cos(\omega t) \right] (2y_0)$

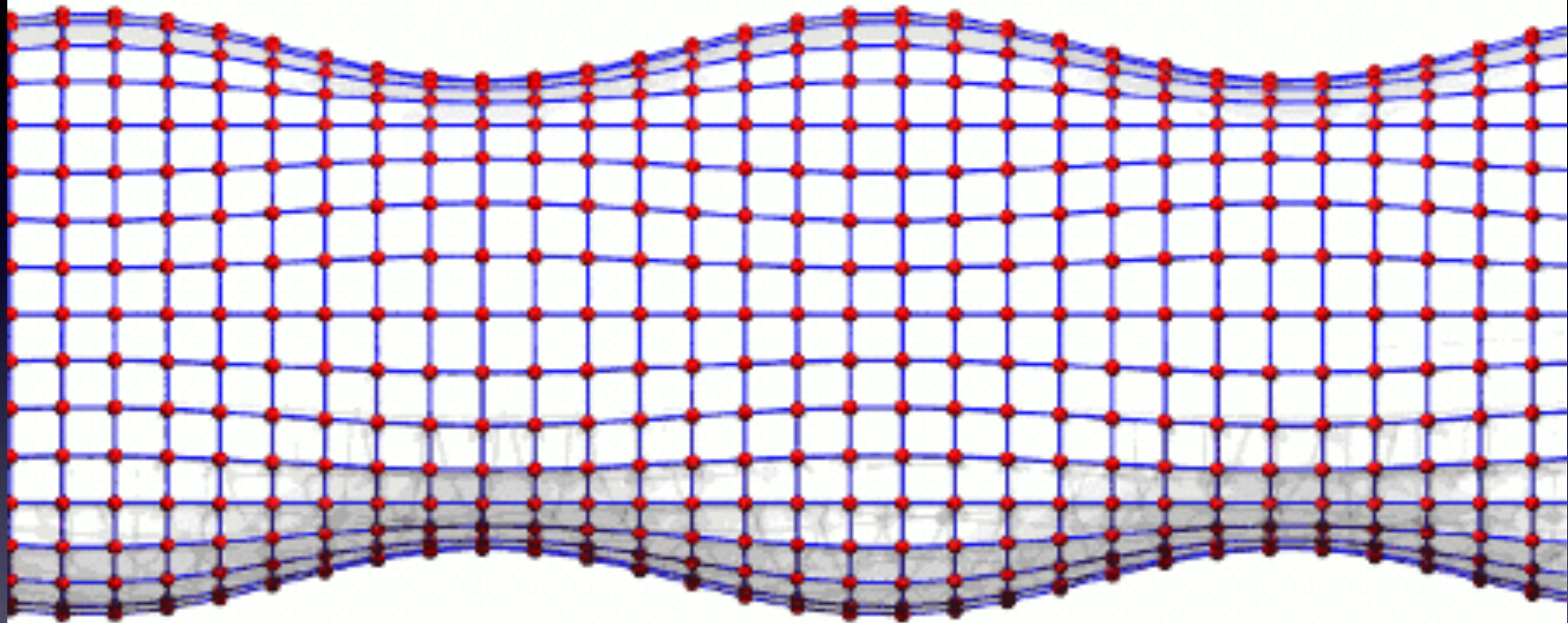


Polarizations present in GR: Fully transverse to the line of propagation



Additional Polarizations not present in GR





+ waves

Basic estimates (I)

- The quadrupole moment of a system is approximately equal to the **mass M** of the part of the system that moves, times the square of the **size R** of the system. This means that the 3rd-order time derivative of the quadrupole moment is:

$$\ddot{\ddot{Q}} \sim \frac{MR^2}{T^3} \sim \frac{Mv^2}{T} \sim \frac{E_{\text{ns}}}{T}$$

v = mean velocity of source's non-spherical motion,

E_{ns} = kinetic energy of non-spherical motion

T = timescale for a mass to move from one side of the system to the other.

- For a self gravitating system: $T \sim \sqrt{R^3/GM}$
- This relation provides a rough estimate of the characteristic frequency of the system $f \sim 2\pi/T$.

GW emission from a binary system (IV)

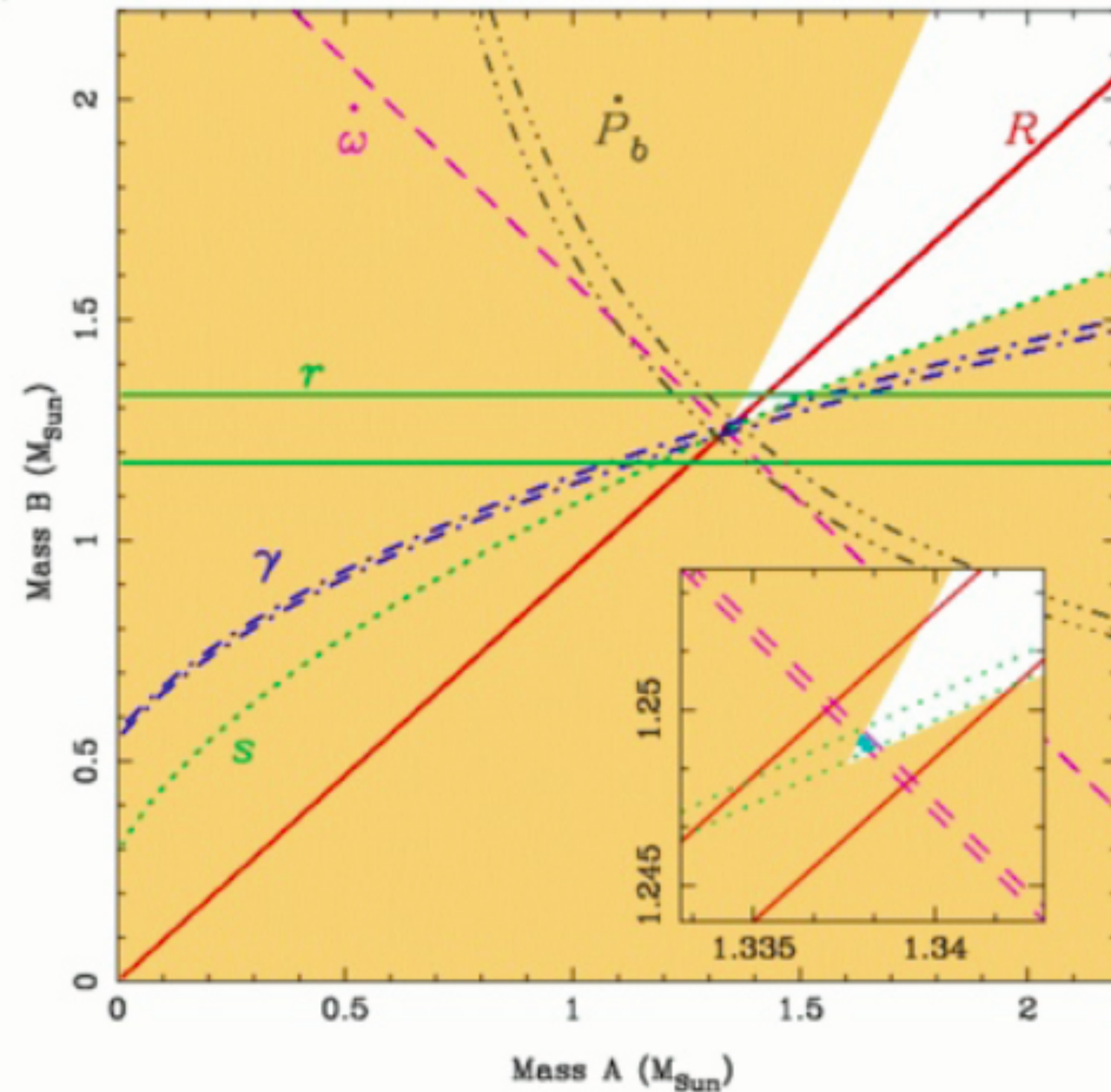
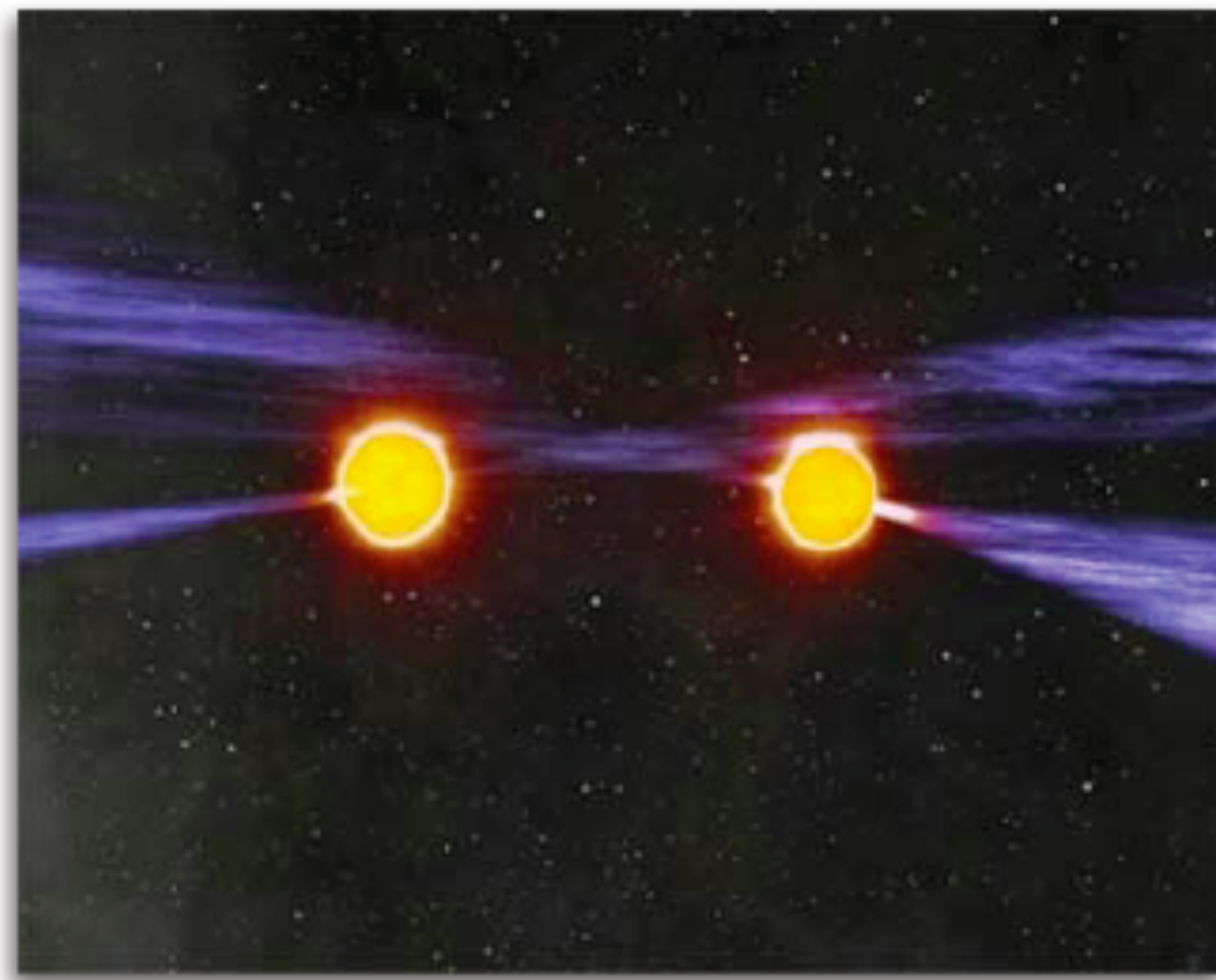
- In this analysis we have assumed circular orbits. In general the orbits can be elliptical, but it has been shown that GW emission **circularizes** them faster than the coalescence timescale.
- The GW amplitude is (ignoring geometrical factors):

$$h \approx 5 \times 10^{-22} \left(\frac{M}{2.8 M_{\odot}} \right)^{2/3} \left(\frac{\mu}{0.7 M_{\odot}} \right) \left(\frac{f}{100 \text{ Hz}} \right)^{2/3} \left(\frac{15 \text{ Mpc}}{r} \right)$$

(set distance to the Virgo cluster, why?)

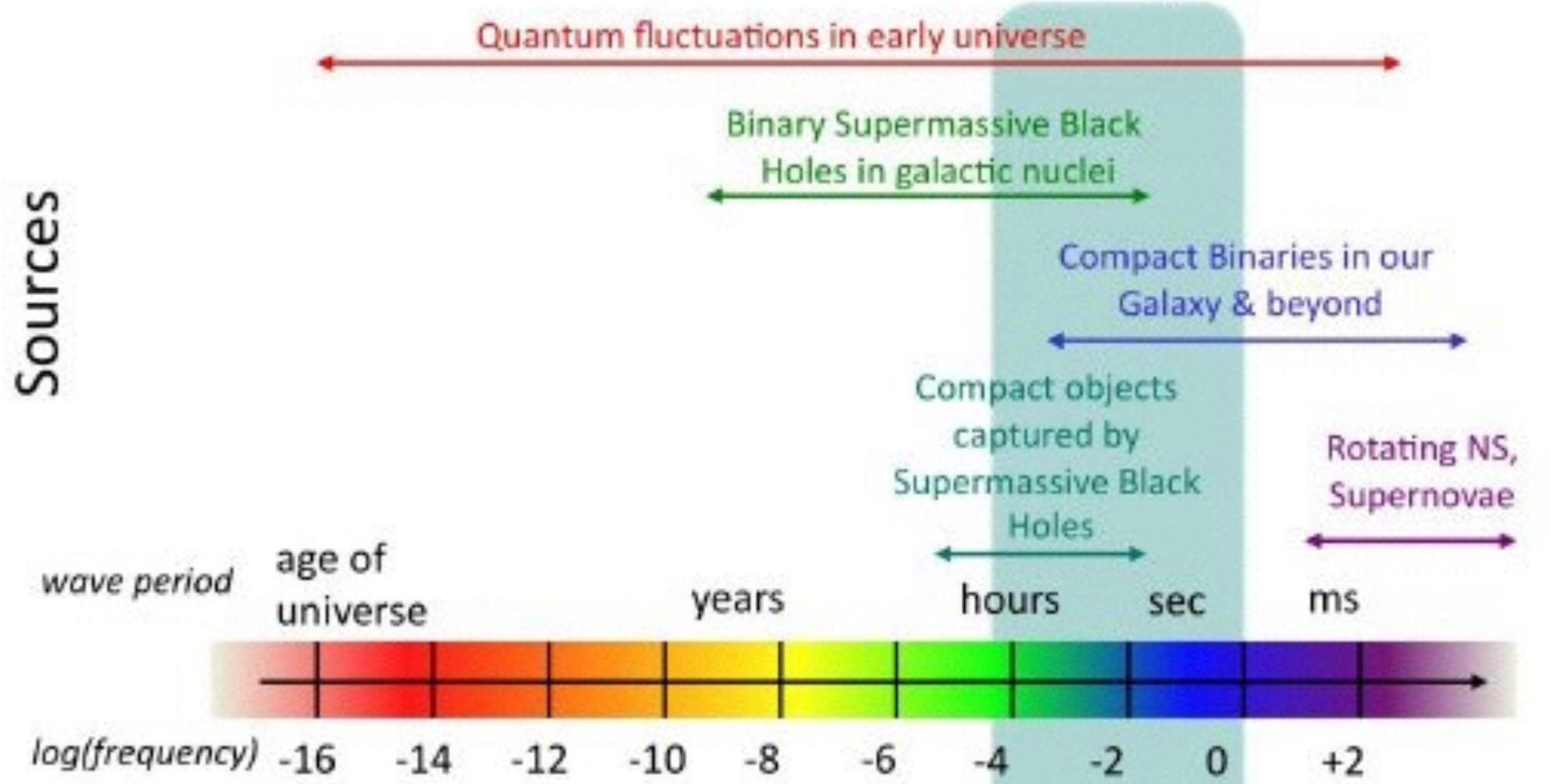
The double pulsar system

- Discovered in 2003, this binary system consists of **two pulsars**: PSR J0737–3039A & B.
- This rare system allows for high-precision **tests of GR**.



The Gravitational Wave Spectrum

Sources



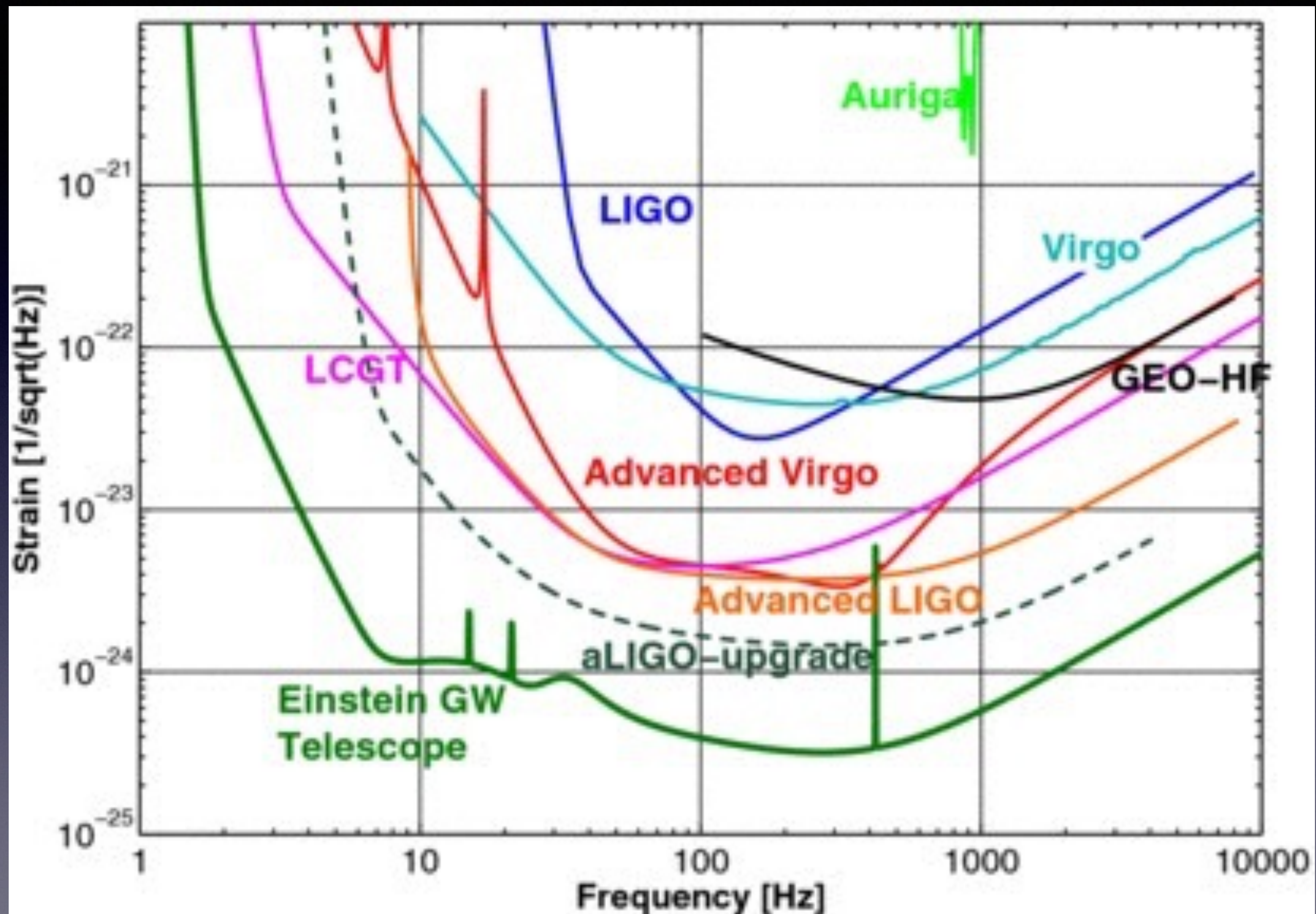
Detectors

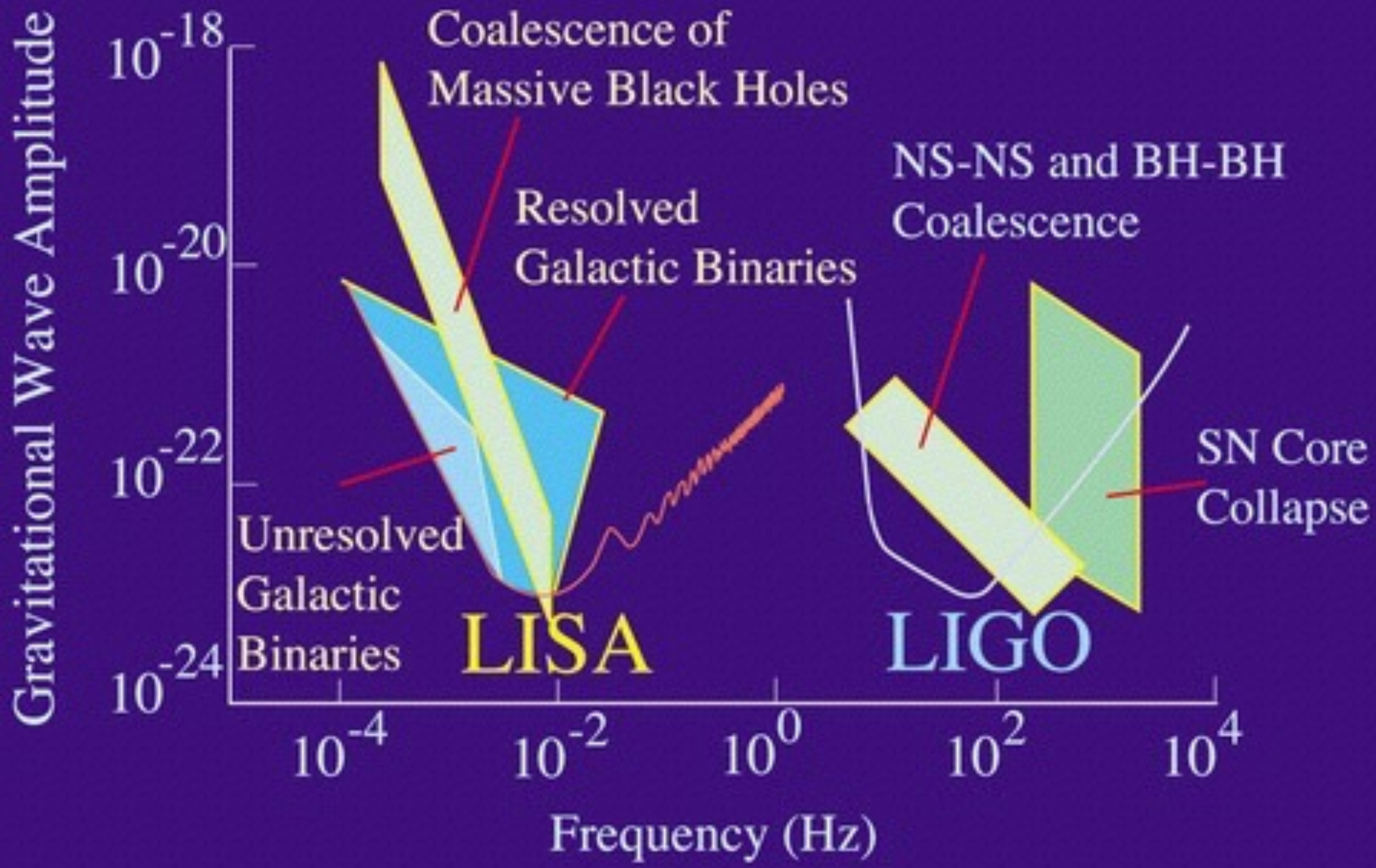


Detectors: the present (II)



The VIRGO detector ($L=3$ km) near Pisa, Italy





Advanced LIGO: By the numbers



4 kilometers

The length of the L-shaped interferometers that contain Advanced LIGO's instrumentation and approximately 40 city blocks in length.

2
laser beams

Actually one that is split into two rays that go back and forth in interferometer vacuum tubes between precisely configured mirrors.

P
1/1000 of a proton diameter

The degree of movement LIGO laser beams could detect in the mirrors; Advanced LIGO is 10 times more sensitive.



< 1 nanosecond after Big Bang

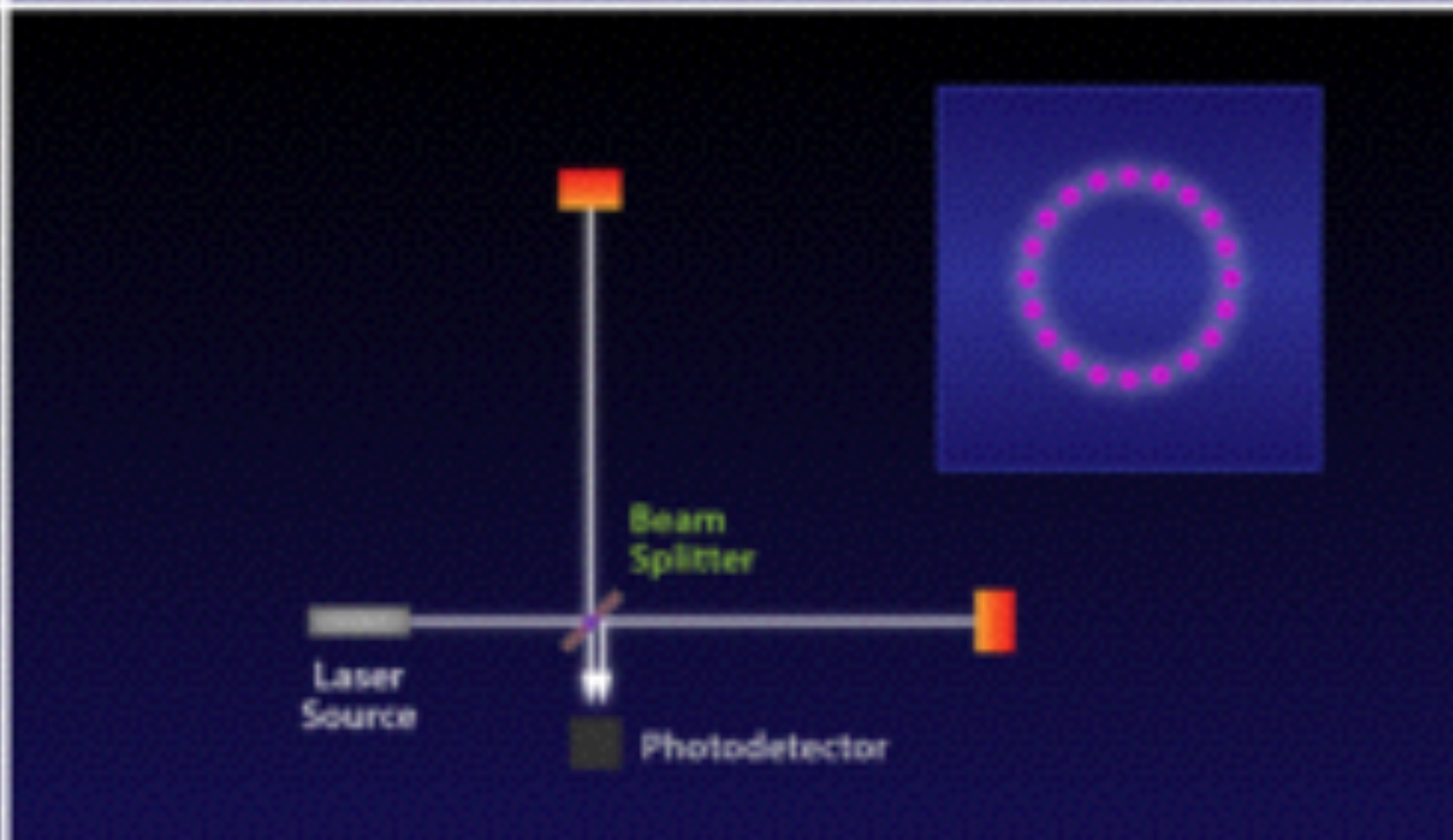
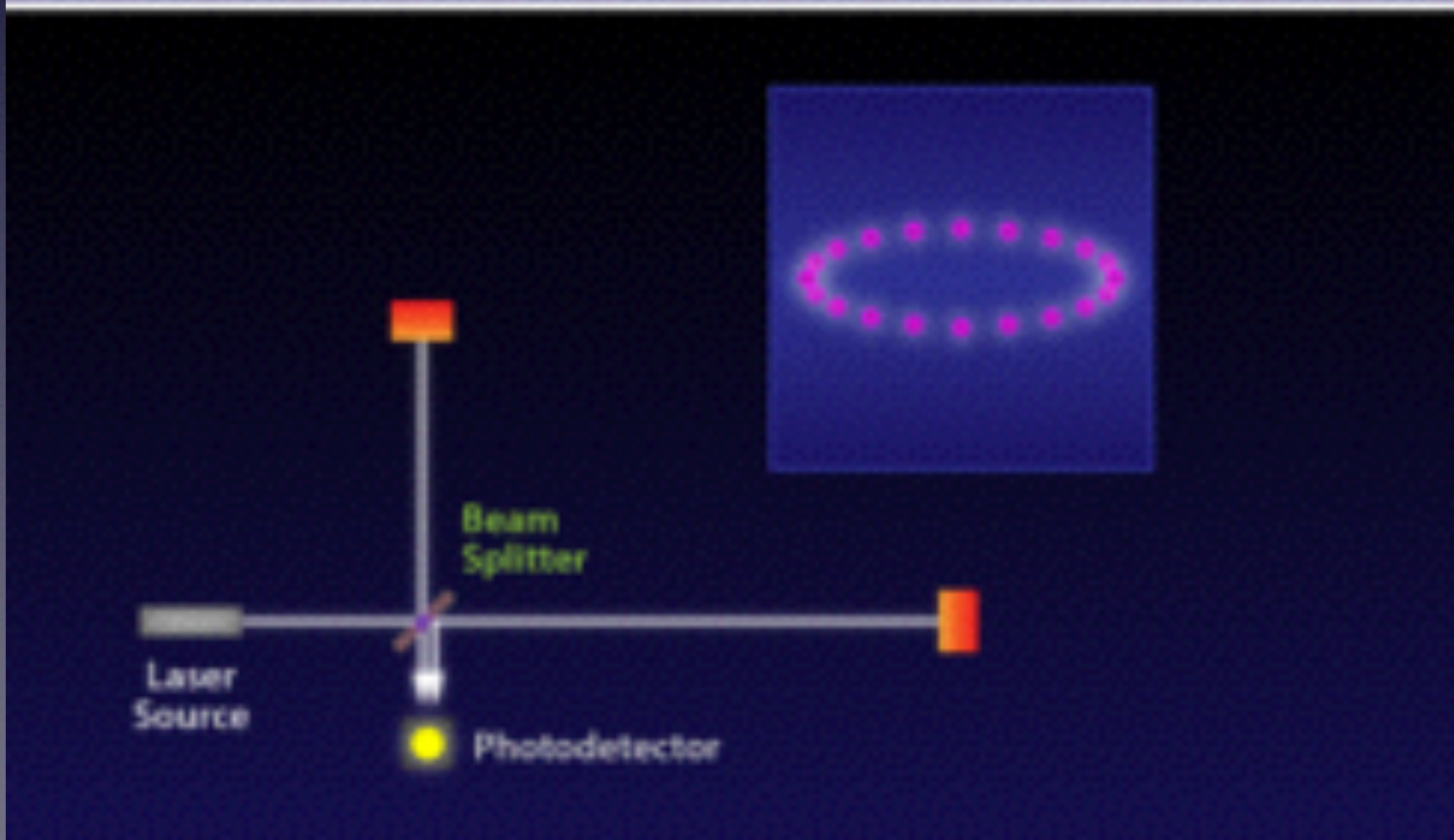
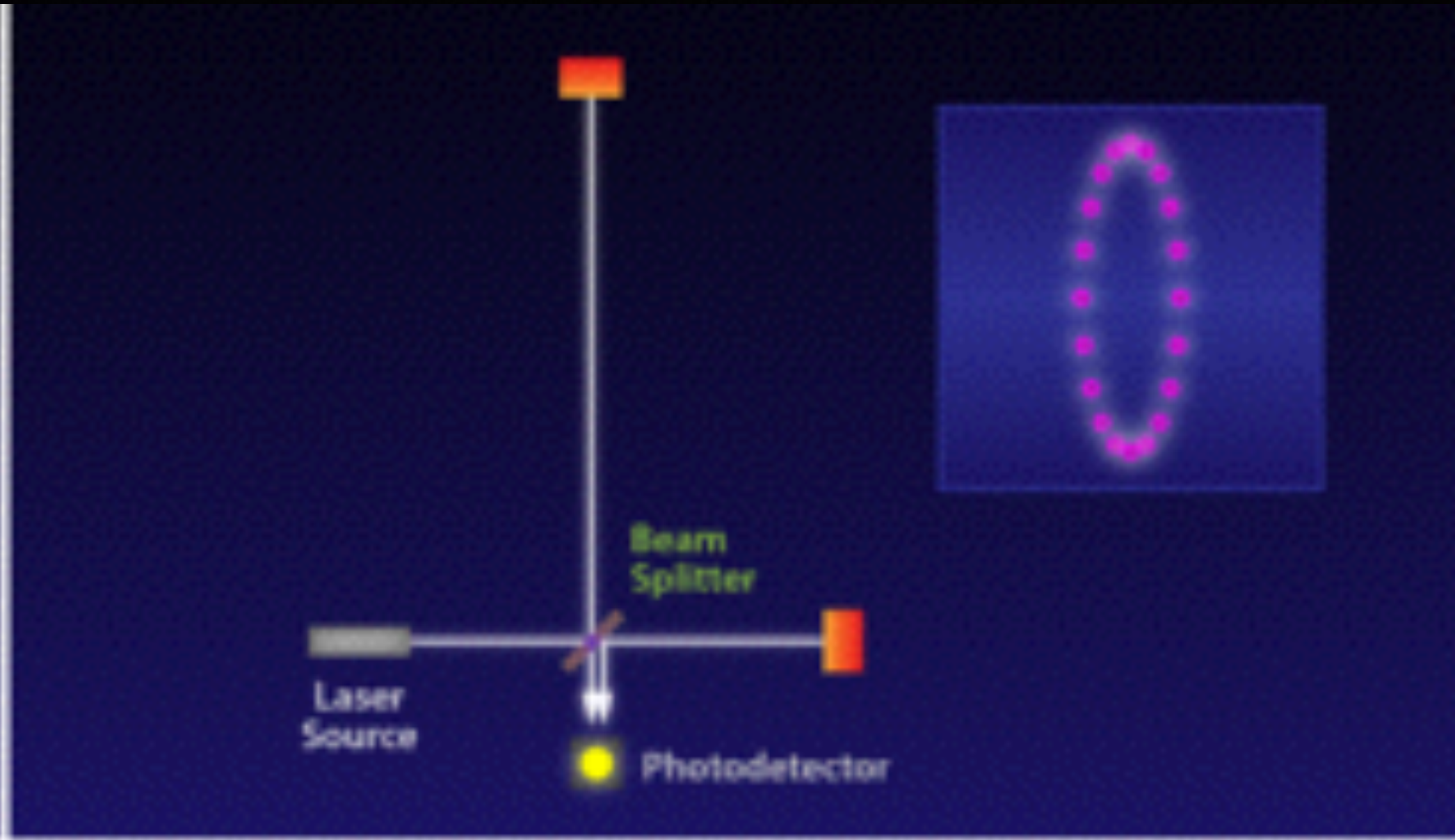
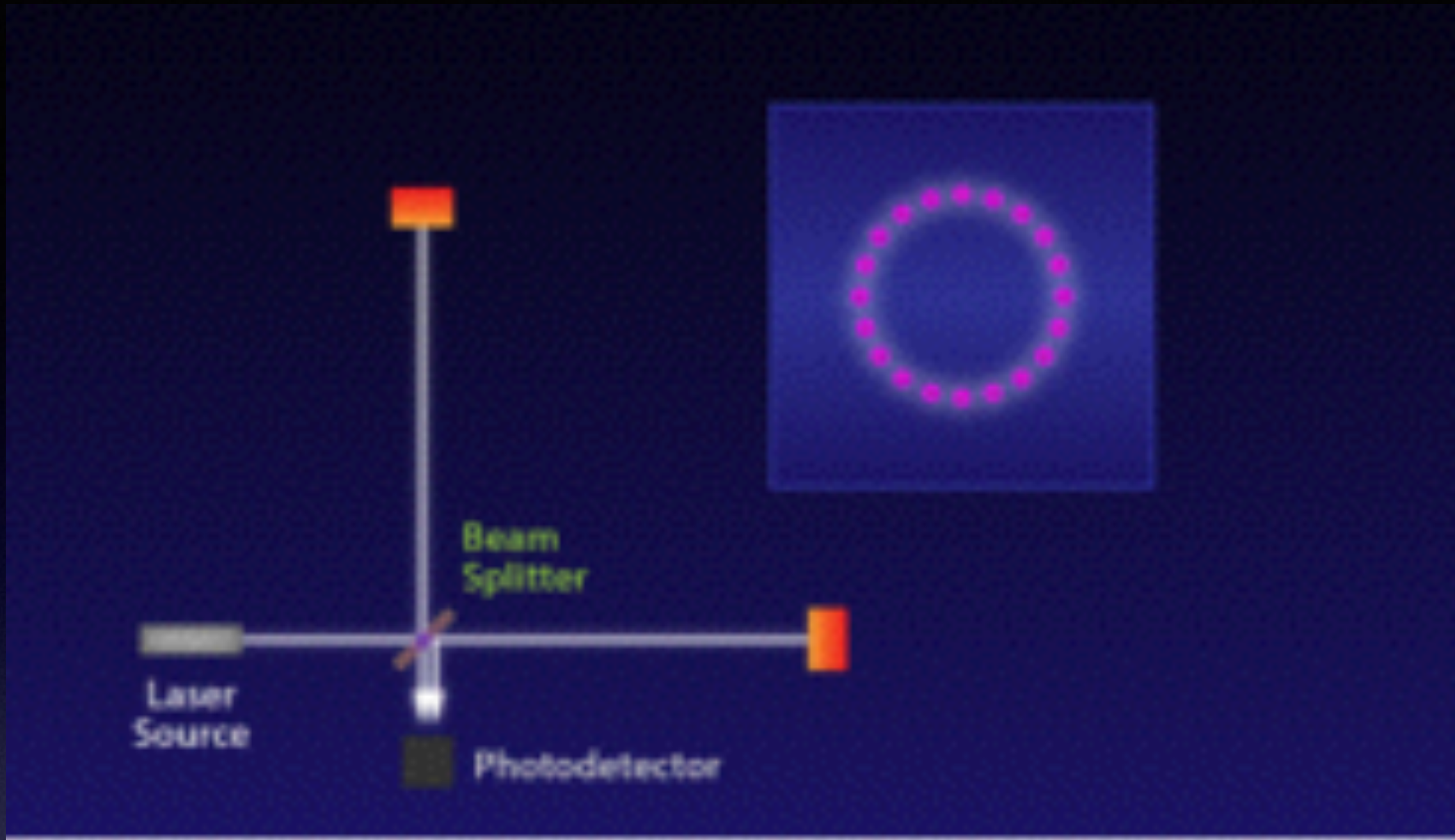
The cosmic gravitational background from this time period that scientists hope to capture to test theories about the universe's development



10-1000Hz

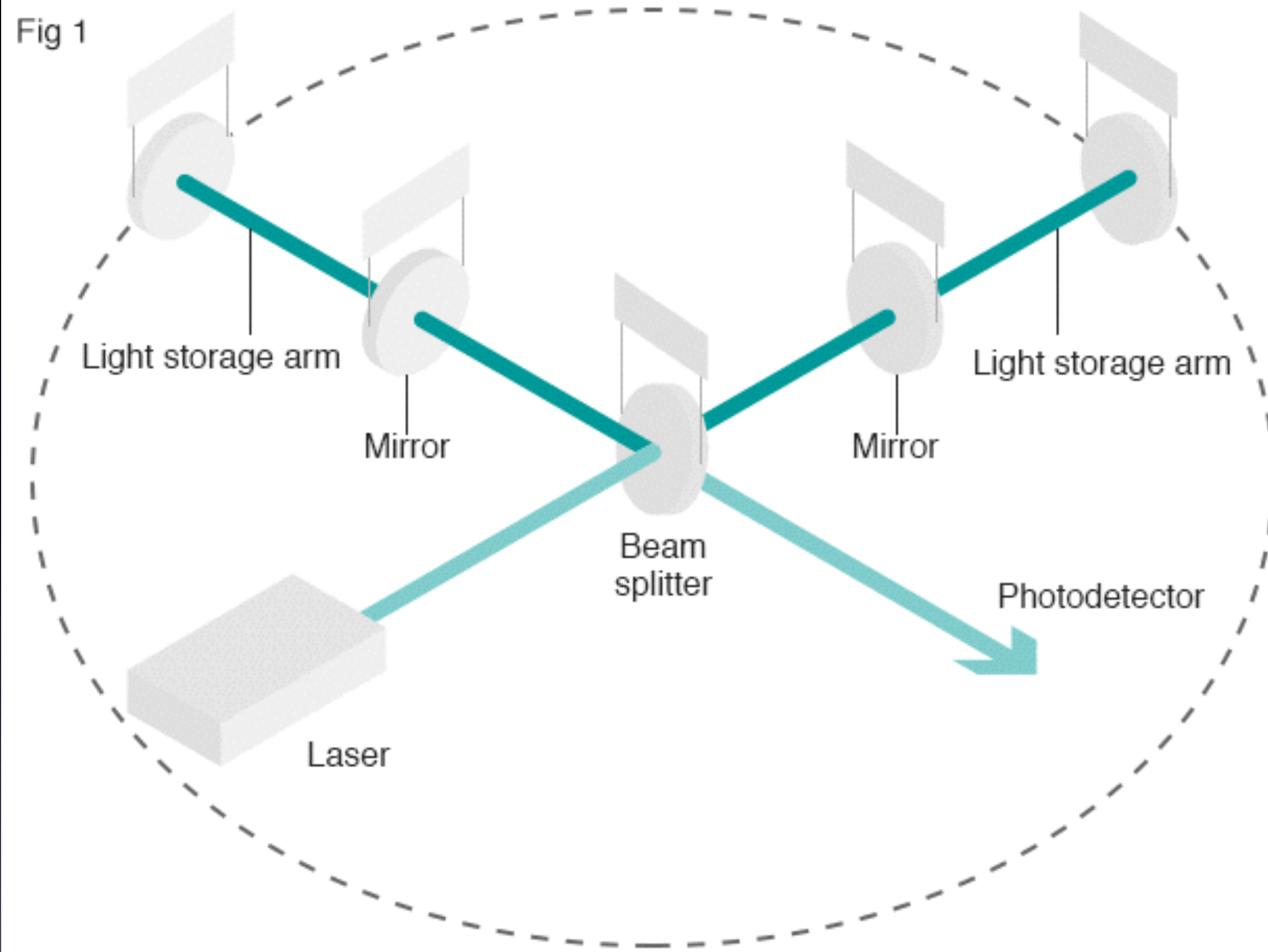
Advanced LIGO's increased frequency range, which is key to observing signals from coalescing black holes and pulsars

The California Institute of Technology and Massachusetts Institute of Technology designed and operate the NSF-funded Advanced Laser Gravitational Wave Observatories (Advanced LIGO) that are aimed to see and record gravitational waves for the first time, allowing us to learn more about phenomenon like supernovae and colliding black holes that propagate these ripples in the fabric of time and space.

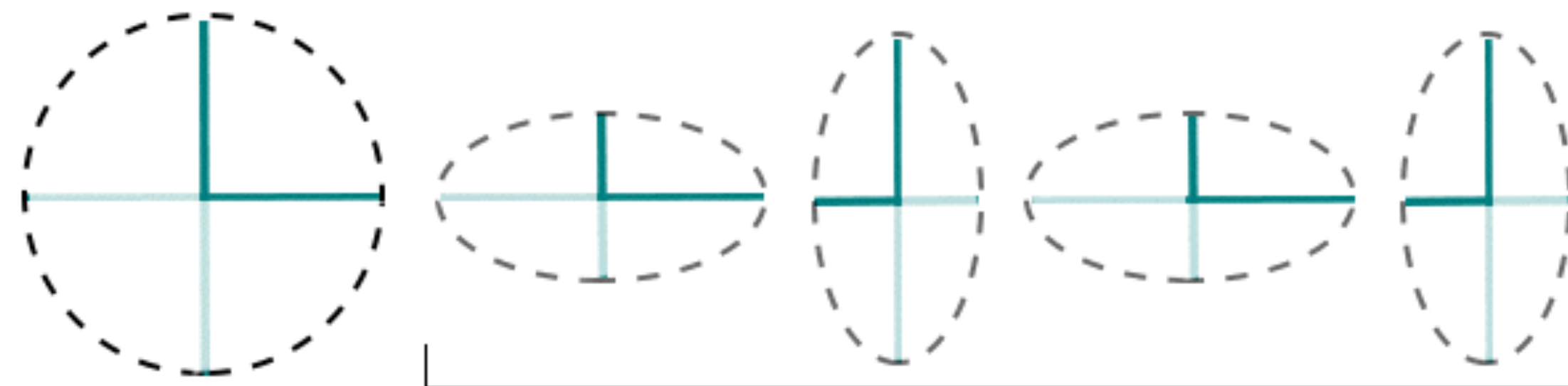


An interferometer: How a gravitational wave hunter works

Fig 1



Gravitational waves alternately stretch and squeeze the space they pass through

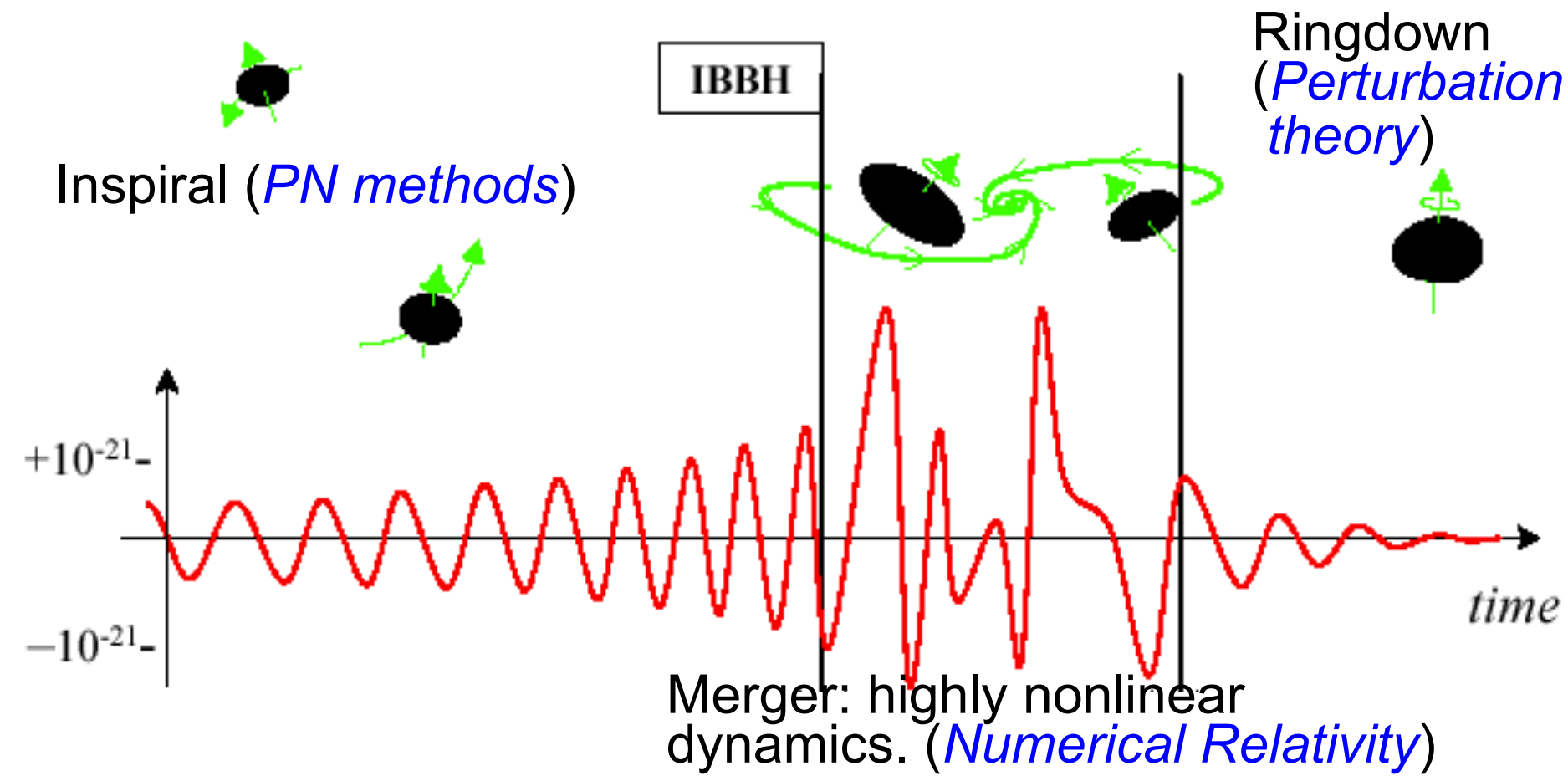


No gravitational waves
(As in Fig 1)

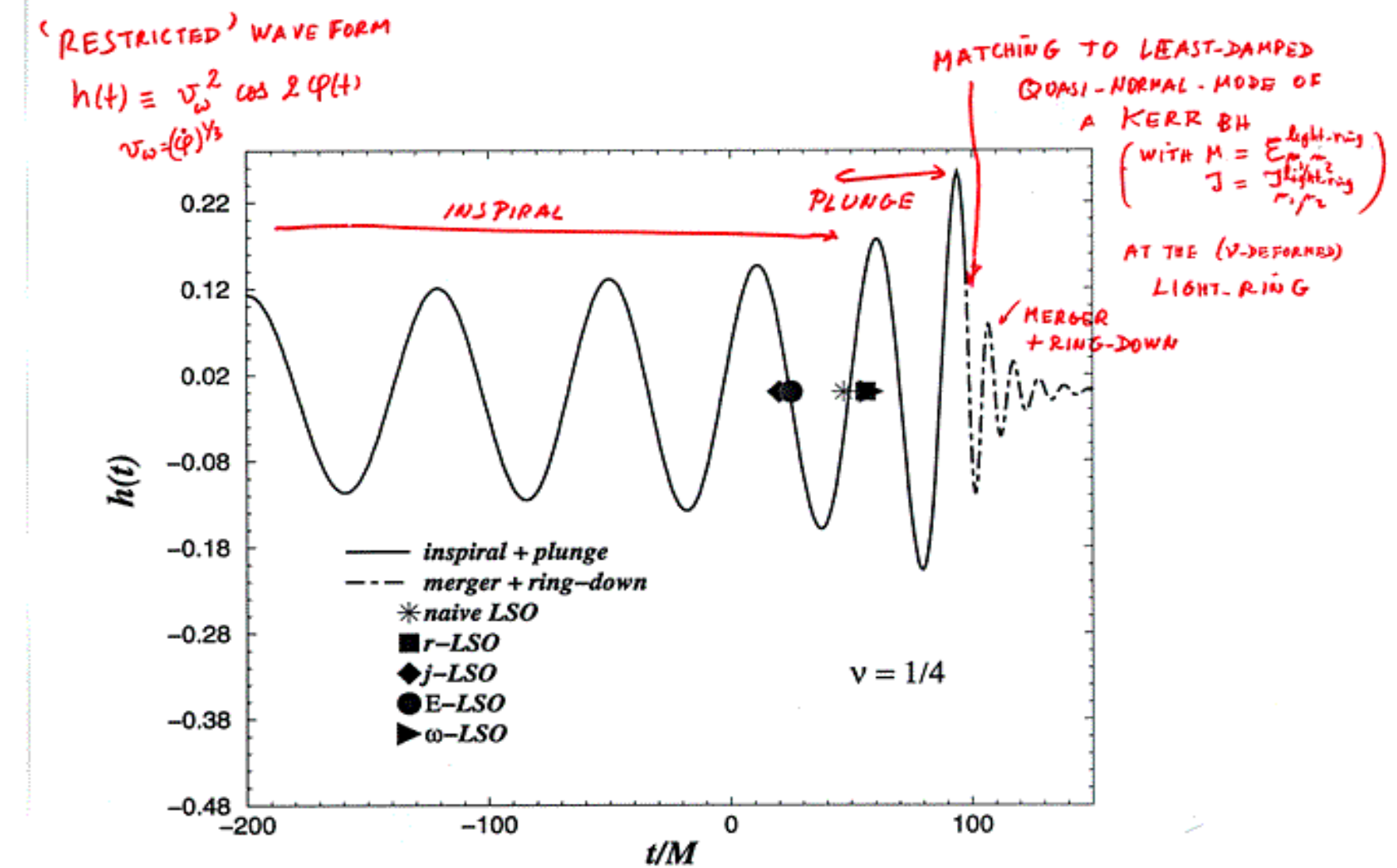
Affected by gravitational waves

Templates for GWs from BBH coalescence

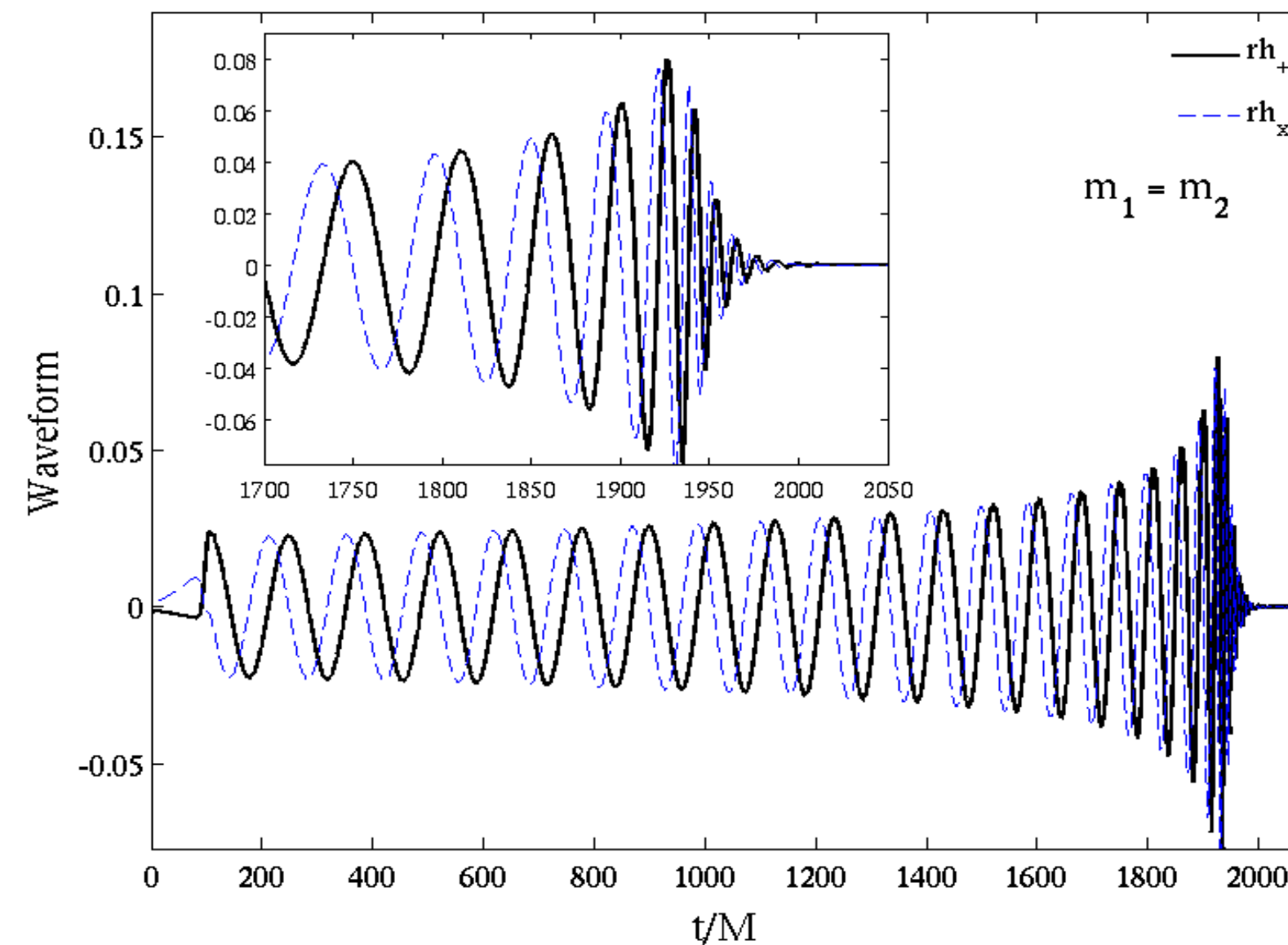
(Brady, Craighton, Thorne 1998)



(Buonanno & Damour 2000)



Numerical Relativity, the 2005 breakthrough:
Pretorius, Campanelli et al., Baker et al. ...



A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy [\[arXiv: 1304.6077\]](https://arxiv.org/abs/1304.6077)

Abdul H. Mroué,¹ Mark A. Scheel,² Béla Szilágyi,² Harald P. Pfeiffer,¹ Michael Boyle,³ Daniel A. Hemberger,³ Lawrence E. Kidder,³ Geoffrey Lovelace,^{4,2} Sergei Ossokine,^{1,5} Nicholas W. Taylor,² Anil Zenginoğlu,² Luisa T. Buchman,² Tony Chu,¹ Evan Foley,⁴ Matthew Giesler,⁴ Robert Owen,⁶ and Saul A. Teukolsky³

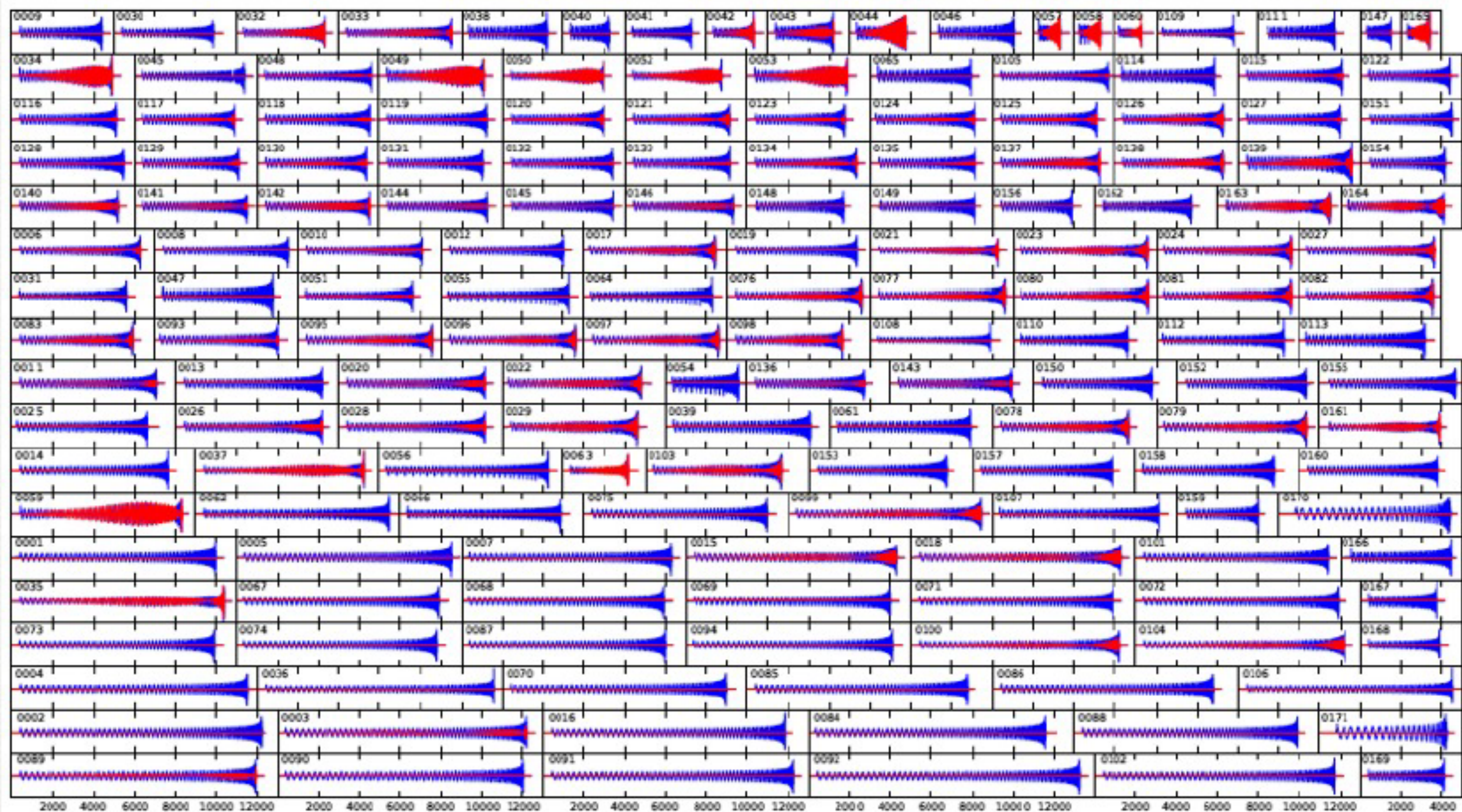
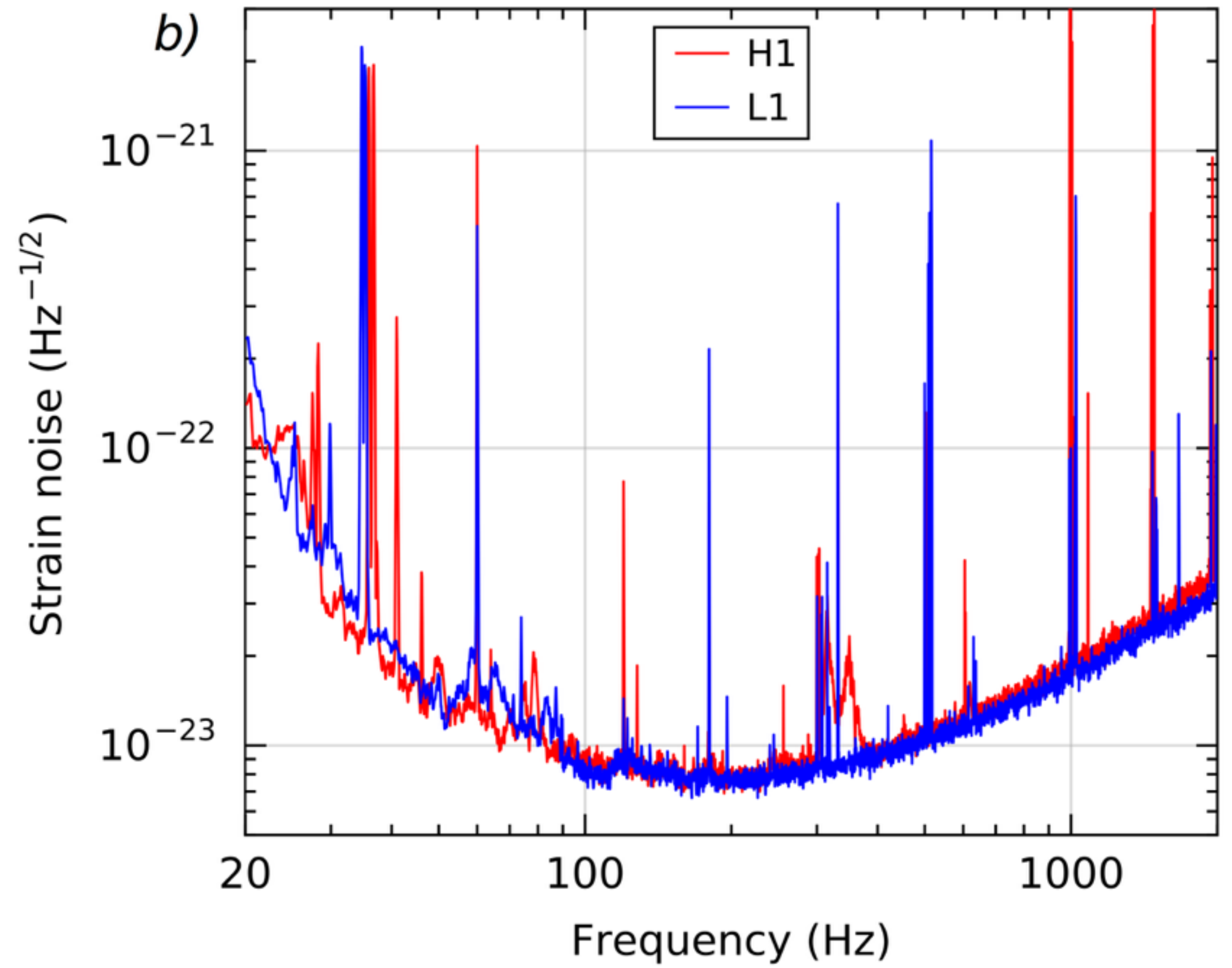
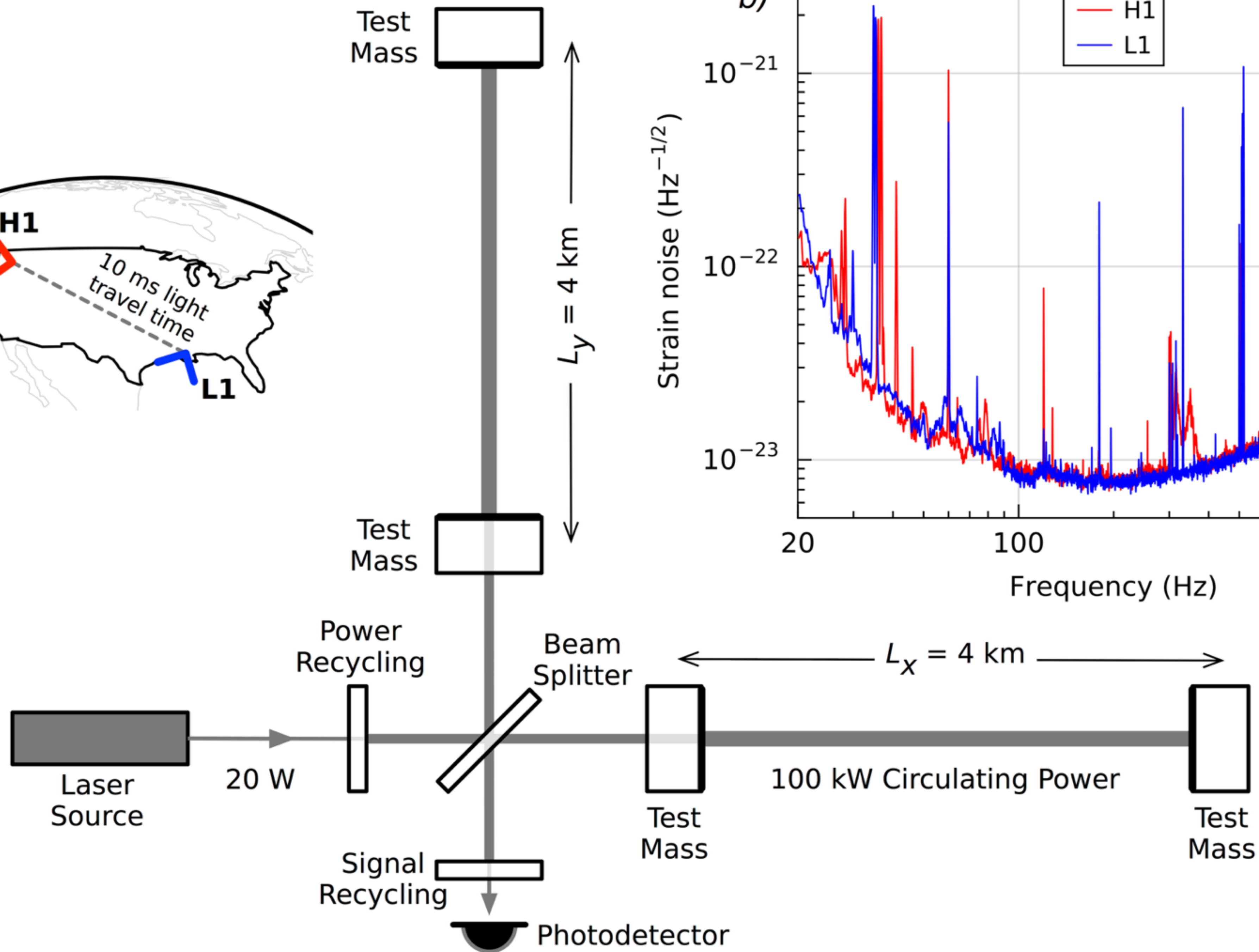
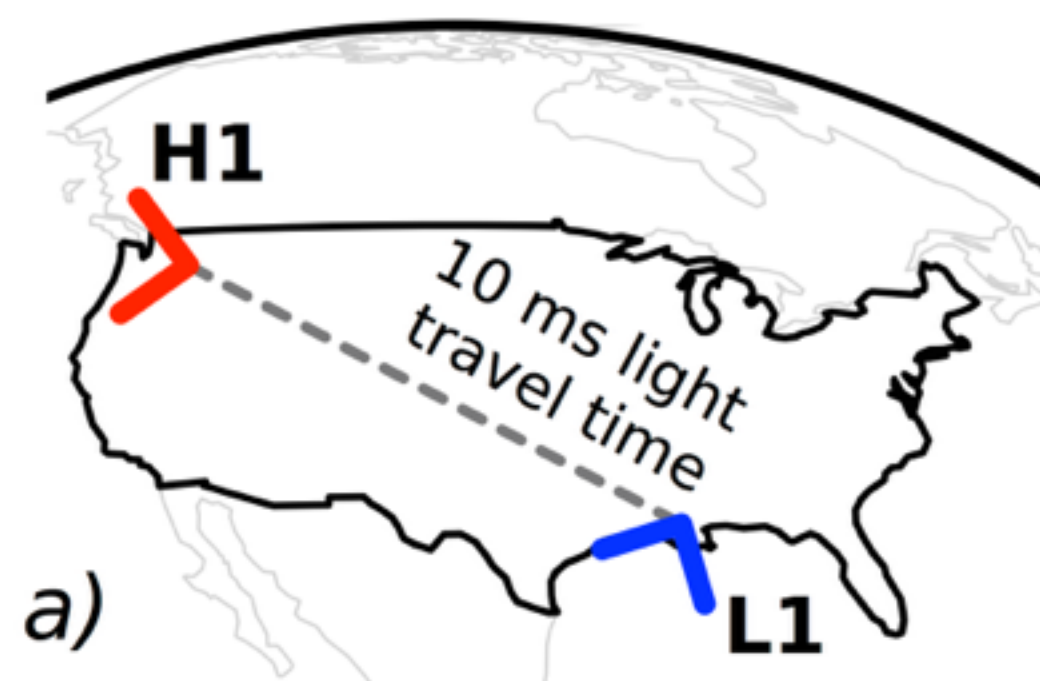
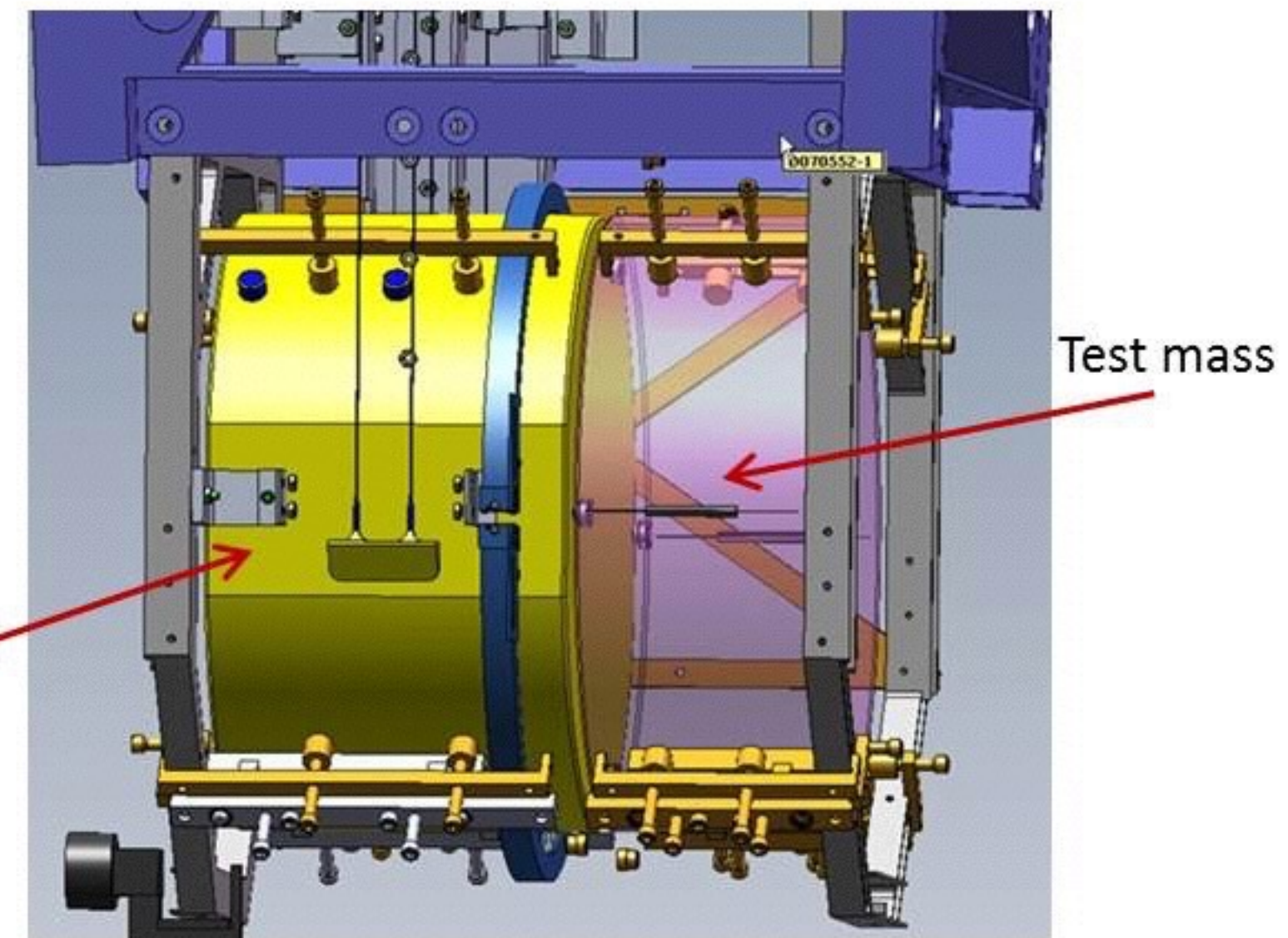
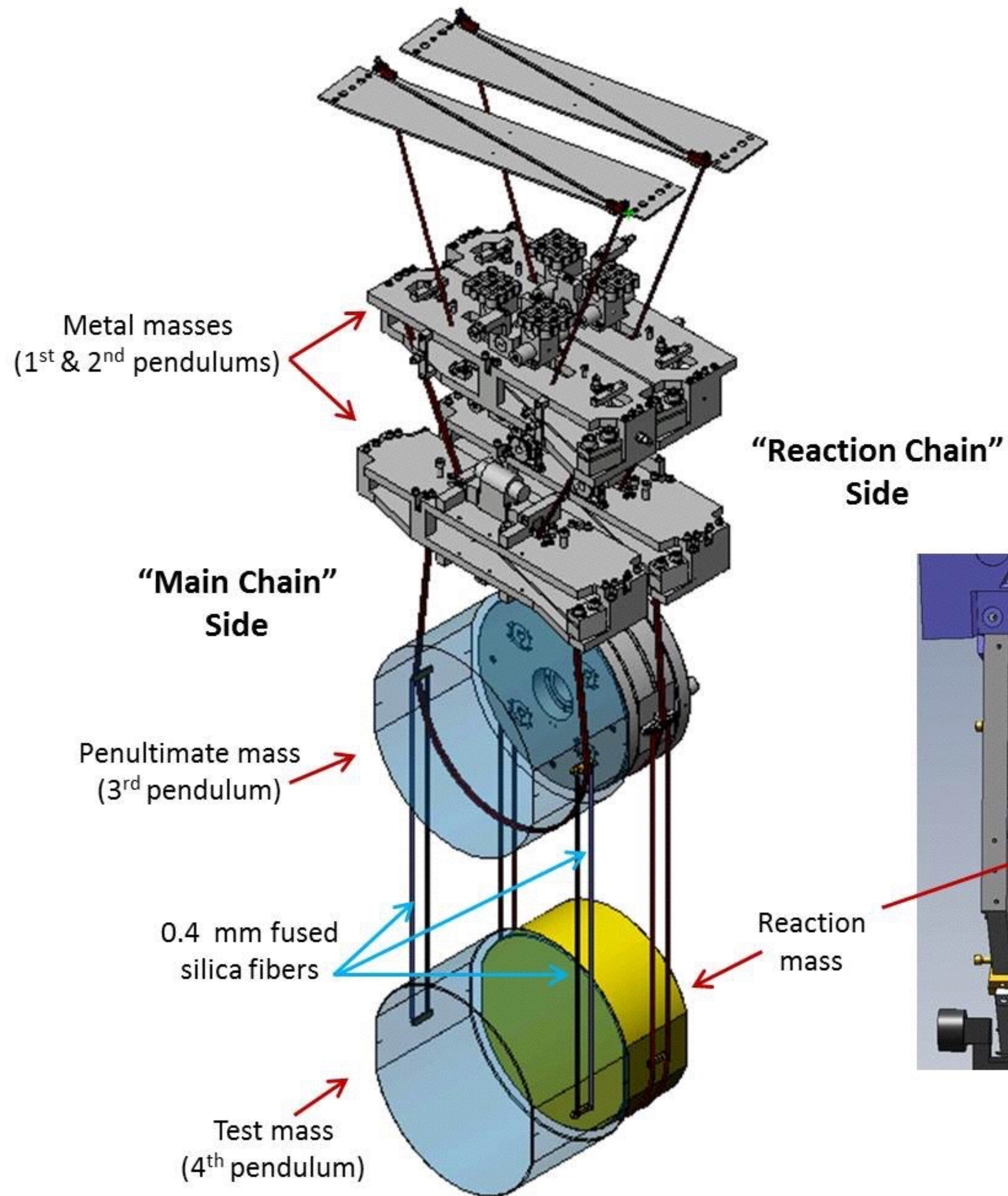


FIG. 3: Waveforms from all simulations in the catalog. Shown here are h_+ (blue) and h_x (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$, where M is the total mass.





Side-view of reaction and test masses

Why spacetime is 4D?

$$R_{\mu\nu} = 0.$$

No. of spacetime dimensions	2	3	4
No. of field equations	3	6	10
No. of independent components of $R_{\mu\nu\sigma\rho}$	1	6	20

Gravitation in empty space can only exist if $n > 3$

