Gravitational lensing and exact geometrical models for astrophysical systems in the cosmological context

talk by: Ezequiel F. Boero
collaborator: Osvaldo M. Moreschi

Arxiv:1610.06032 , Arxiv:1611.05832

Fof-Meeting - 07 April 2017 - Córdoba

UNC Universidad Nacional de Córdoba
FAMAF Facultad de Matemática, Astronomía, Física y Computación

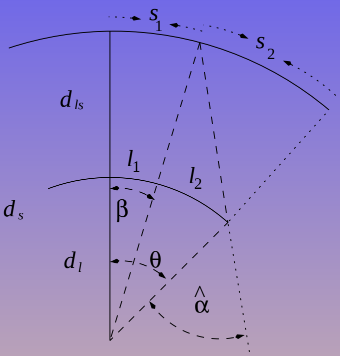
Outline

- The usual formalism of lenses is based in a linear superposition of Schwarzschild masses

$$\alpha(\xi) = \frac{4M}{\xi}, \qquad \alpha(\vec{\xi}) = \sum_i 4m_i \frac{\vec{\xi} - \vec{\xi}_i}{\left|\vec{\xi} - \vec{\xi}_i\right|^2} \tag{1}$$

  however it may be useful to consider general situations where the whole energy-momentum of a continuous distributions is taken into account.
- Some works have suggested that a broader approach in the mass content may be useful in the description of DM.
- In particular, it is desirable to have at hand expressions in terms of the curvature of the lens and not in terms of metric functions.
- Our motivations is improve in standard tools from first principles (deviations geodesic equation) to obtain general expressions in a cosmological context.
- This would allow to deal with geometrical models of the lens.
- We present a novel formalism for weak lensing and simple geometrical models for describing some astrophysical systems.

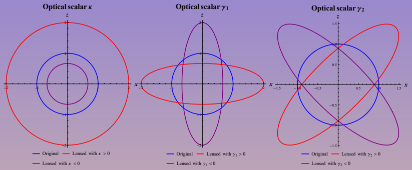
Angular variables



Optical scalars

$$\delta\beta^a = \mathcal{A}^a_{\ b}\,\delta\theta^b$$

$$\mathcal{A}^a_{\ b} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$



- We make the comparison with respect two spacetime and we retain the basic notions:
  - θ is the observed angle (i.e. non-trivial curvature)
  - β is the angle that one would observe without lens (i.e. no curvature).

- It is crucial to establish the distance to the source: the unique well defined geometrical distance is the affine distance, namely λ.

Distances

- For an object of size δl one has:

$$\delta l = D_A(\eta)\delta\beta, \tag{2}$$

$$\delta l = D_A(g)\delta\theta; \tag{3}$$

in order to make the comparison one takes $\lambda = \lambda_s$.

The geodesic deviation equation

- With respect to a null tetrad adapted to the path of the photons $(\ell^a, m^a, \bar{m}^a, n^a)$ the deviation geodesic vector $\varsigma^a \equiv \delta l^a$ is:

$$\varsigma^a = \varsigma\bar{m}^a + \bar{\varsigma}m^a, \tag{4}$$

- It satisfies:

$$\boxed{\ell\left(\ell\left(\boldsymbol{\mathcal{X}}\right)\right) = -Q\boldsymbol{\mathcal{X}},} \tag{5}$$

$$\boldsymbol{\mathcal{X}} = \begin{pmatrix} \varsigma \\ \bar{\varsigma} \end{pmatrix} \quad \text{and} \quad Q = \begin{pmatrix} \Phi_{00} & \Psi_0 \\ \bar{\Psi}_0 & \Phi_{00} \end{pmatrix}. \tag{6}$$

In a Robertson-Walker cosmology

- The lens is actually the whole spacetime; i.e. it is not placed a any particular distance.
- Due to the symmetry there is no notion of deflection angle.
- Howerver, there exist a notion for the optical scalars:

$$\delta\beta = (1-\kappa_c)\delta\theta. \tag{7}$$

Magnifications and magnitudes

- Angular magnification μ:

$$\mu(\lambda) = \frac{1}{(1-\kappa)^2 - (\gamma_1^2+\gamma_2^2)} \qquad \text{let us note that} \qquad \mu \geq 1. \tag{8}$$

- Intensity magnification $\tilde{\mu}$:

$$\tilde{\mu}(\lambda) = \frac{\mathscr{F}(\lambda, z)}{\mathscr{F}_0(\lambda, z)}; \tag{9}$$

due to Etherington's theorem one has

$$\boxed{\mu = \tilde{\mu}} \tag{10}$$

- Cosmological intensity magnification μ'_c:

$$\mu'_c(z) = \frac{\mathscr{F}(z)}{\mathscr{F}_{\text{Milne}}(z)}; \tag{11}$$

is only function of redshift as usually in practical applications.
It recurs to the flat Milne Universe where one has a relation $\lambda(z)$.

- The above magnification are related to the astronomical magnitudes.
One can consider two different meanings for the magnitudes:

$$m - M = -\frac{5}{2}\log\frac{\mathscr{F}(\lambda, z)}{\mathscr{F}(\lambda_{10}, z_{10})} = -\frac{5}{2}\log\mu(\lambda) + 5\log\left(\frac{\lambda(1+z)^2}{\lambda_{10}}\right)$$

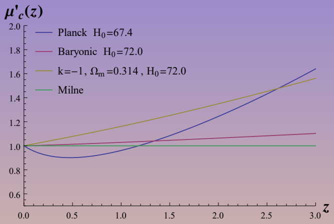
it is valid in any spacetime and only contains information of the optical scalars and kinematic variables.

When the redshift is employed as indicator of distances one has:

$$m - M = -\frac{5}{2}\log\frac{\mathscr{F}(\lambda(z),z)}{\mathscr{F}(\lambda_{10}(z_{10}),z_{10})} = -\frac{5}{2}\log\mu'_c(z) + 5\log\left(\frac{\lambda_{\text{Milne}}(z)(1+z)^2}{\lambda_{10}}\right)$$

it only can be used when one has additional structure on the spacetime such as in cosmology where a family of preferred observed is present.

- Let us note that $\mu'_c(z)$ is the quantity intervening in the usual supernovae analysis where a cosmological constant Λ is argued.



Let us note that for the concordance model $\mu'_c \leq 1$.

- However, if one utilize the notion $\tilde{\mu} = \mu$ one always would obtain $\mu \geq 1$.
- At the same time, the fluxes does not contain information about a cosmological constant Λ because μ contains the traceless part of the Ricci and Weyl tensors.

An additional lens over the cosmology

- An alteration of the complete homogeneity and isotropy in scales much smaller than the cosmological ones.
- The optical scalars are computed without recurring to the deflection angle

$$\ell\left(\ell\left(\boldsymbol{\mathcal{X}}\right)\right) = -\left(Q_B + \delta Q\right)\boldsymbol{\mathcal{X}}, \tag{12}$$

where

$$\delta Q = \begin{pmatrix} \delta\Phi_{00} & \delta\Psi_0 \\ \delta\bar{\Psi}_0 & \delta\Phi_{00} \end{pmatrix}. \tag{13}$$

New expression for the optical scalar in the cosmological context

- In the way we are working the cosmological contributions to the optical scalars appear explicitly:

$$\kappa = (1-\kappa_c)\,\kappa_L + \kappa_c, \tag{14}$$

$$\gamma_1 = (1-\kappa_c)\,\gamma_{1L}, \tag{15}$$

$$\gamma_2 = (1-\kappa_c)\,\gamma_{2L}. \tag{16}$$

- The optical matrix has the following structure

$$\mathcal{A} = (1-\kappa_c)\begin{pmatrix} 1-\kappa_L-\gamma_{1L} & -\gamma_{2L} \\ -\gamma_{2L} & 1-\kappa_L+\gamma_{1L} \end{pmatrix}; \tag{17}$$

where the intrinsic optical scalars κ_L, γ_{1L}, γ_{2L} are given by the generalized expressions

$$\kappa_L = \int_0^{\lambda_s} \left(\frac{1}{D_A^2} \int_0^{\lambda'} \delta\Phi_{00}\, D_A^2\, d\lambda'' \right) d\lambda', \tag{18}$$

$$\gamma_{1L} = \int_0^{\lambda_s} \left(\frac{1}{D_A^2} \int_0^{\lambda'} \delta\Psi_{0R}\, D_A^2\, d\lambda'' \right) d\lambda', \tag{19}$$

$$\gamma_{2L} = \int_0^{\lambda_s} \left(\frac{1}{D_A^2} \int_0^{\lambda'} \delta\Psi_{0I}\, D_A^2\, d\lambda'' \right) d\lambda'. \tag{20}$$

They contains the whole curvature and therefore the complete energy-momentum tensor of the lens.

In the general case one has to take into account that κ_L y γ_L are independent.

The thin lens approximation

- The size of the lens is smaller than the distances between the source and lens and between lens and observer.
- The optical scalars simplify to

$$\circ_L = \mathbf{D}_{ls} \int_0^{\lambda_s} \delta C \, d\lambda', \tag{21}$$

where we denote generically the curvature as $\delta C = \{\delta\Phi_{00}, \delta\Psi_0\}$.

- The distance factor $\mathbf{D}_{ls}$ only contains information about the cosmology:

$$\mathbf{D}_{ls} = \frac{1}{1+z_l} \frac{D_{A_{ls}} D_{A_l}}{D_{A_s}}. \tag{22}$$

- For a moving lens one can express the optical scalars in terms of the of the rest frame of the lens.
  This frame is related to the frame of the fundamental observers by boost

Weak lensing in cosmology X

- Transformation properties of the curvature scalars show that both behave in the same way under boost:

$$\circ_{Lv} = (1 + z_v)\circ_{Lr}, \tag{23}$$

  Comparing with the usual expressions one has

$$\circ_{Lv} = \frac{1 + z_v}{1 + z_l} \frac{D_{A_{ls}} D_{A_l}}{D_{A_s}} \int_0^{\lambda_s} \delta C_r \, d\lambda'_r \tag{24}$$

  where the part in blue correspond to the usual quotient $\frac{\Sigma}{\Sigma_{cr}}$.
- The corrections is very small in typical situations since peculiar velocities small.
- However, the derivations of the motion of the lens is straightforward in this approach.

- The formalism invite us to work with exact geometrical models.
  A broader approach than just consider the energy density of the lens.

$$\kappa_L\left(J\right) = \frac{4\pi G}{c^2}\mathbf{D}_{ls}(1+z_v)\int_{-\infty}^{\infty}\left[\varrho(\mathbf{r}) + \frac{P_r(\mathbf{r})}{c^2} + \frac{J^2}{c^2\mathbf{r}^2}\Big(P_t(\mathbf{r}) - P_r(\mathbf{r})\Big)\right]dy,$$

$$\gamma_L\left(J\right) = \frac{G}{c^2}\mathbf{D}_{ls}(1+z_v)\int_{-\infty}^{\infty}\frac{J^2}{\mathbf{r}^2}\left[\frac{3M(\mathbf{r})}{\mathbf{r}^3} - \left(\varrho(\mathbf{r}) + \frac{P_t(\mathbf{r})}{c^2} - \frac{P_r(\mathbf{r})}{c^2}\right)\right]dy;$$

- If the spacelike components of the energy-momentum tensor can be important in the description of the matter content then, one would like to have at hand useful metrics to study in a simpler way interesting systems.
- Models in which the energy-content can be controlled in a simple way.
- We consider spheroidal symmetries (prolates and oblates)

Geometries with spheroidal symmetry

- **Prolate geometries**

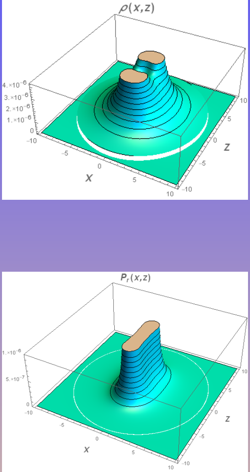
$$ds^2 = a(r)dt^2 - \left(r^2 + r_\mu^2 \sin^2(\theta)\right)\left(\frac{dr^2}{r^2 - 2rM(r) + r_\mu^2} + d\theta^2\right) - r^2 \sin^2(\theta) d\phi^2$$

- **Oblate geometries**

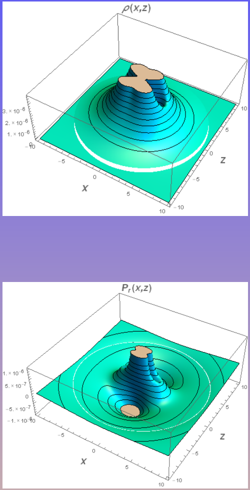
$$ds^2 = a(r)dt^2 - \left(r^2 + r_\mu^2 \cos^2(\theta)\right)\left(\frac{dr^2}{r^2 - 2rM(r) + r_\mu^2} + d\theta^2\right) - \left(r^2 + r_\mu^2\right) \sin^2(\theta) d\phi^2$$

- The associated energy-momentum tensor can be described in terms of three functions $\varrho(r,\theta)$, $P_t(r,\theta)$ and $P_r(r,\theta)$, in complete analogy with the spherical case.
- One recover the more general spherical geometries in the limit $r_\mu \to 0$.

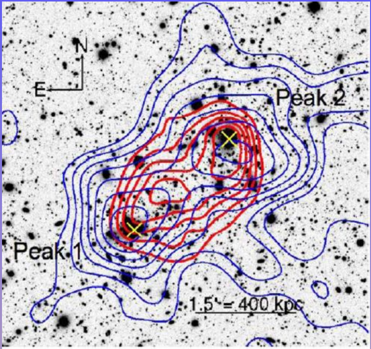
Geometry with an isothermal mass profile: $M(r) = \frac{M_*}{r_*} r$

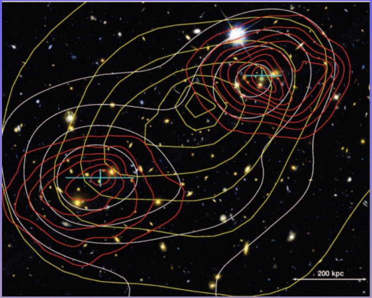


(a) Prolate geometry.



(b) Oblate geometry.

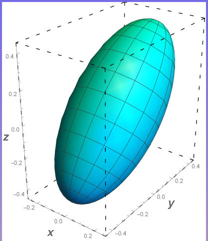


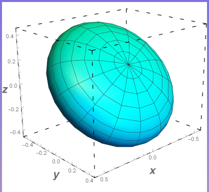


- Clearly some similarities are observed.

Weak lensing in spheroidal geometries

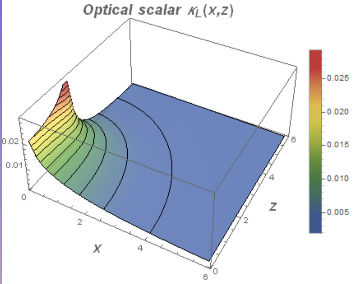
- One must to take into consideration the tilted angle ι

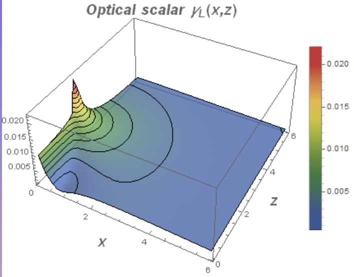




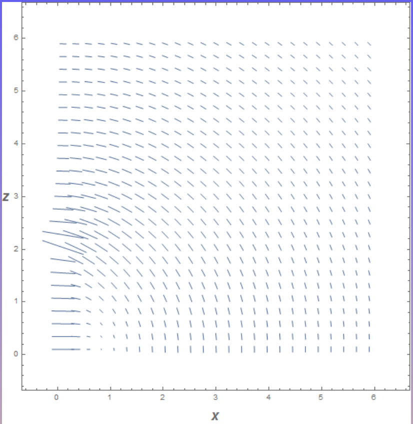
- Analitic expression can be found but optical scalars are better analysed by numerical integration.

Geometry without mass





- A complex behaviour arise because the shear modulus does not copy the projected geometry on the lens plane.



- However shear maps follows the symmetry of the energy-momentum distribution of the lens.

Summary and final comments I

- We have been presented a general formalism for weak lensing immersed in a standard cosmological context.
  The notion of deflection angle can be avoided and the motion of the lens is contained in a very straightforward way.
- The formalism is well suitable for dealing with geometrical models, i.e. the lens is properly described by its metric g_{lens} rather than by just its energy density.
  It implies that one can consider the whole curvature and therefore the whole energy-momentum intervening in the field equations.
- We have pointed out that if the affine parameter is used an slightly different notion of magnification, namely $\tilde{\mu}(\lambda)$ could be considered which is more geometric.
  Since it is expressed in terms of the optical scalars a cosmological constant does not intervene in observations of fluxes.
- We have constructed spheroidal geometries (oblates and prolates) which can be used as simple models for studying astrophysical systems, mainly clusters and voids.

Thank you for your attention!