Skylight: a new code for general relativistic ray tracing and radiative transport in arbitrary spacetimes

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I F E G

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Abstract Modelling the observed spectra and light curves originated in the vicinity of compact objects requires accurate numerical codes for relativistic ray tracing and radiative transfer. Here, we present Skylight, a new code we developed for achieving such purposes in arbitrary space-time geometries. From the code we can extract images, spectra and light curves as seen by distant observers starting from astrophysical models of the compact sources. The code can operate under two different schemes, namely a Monte Carlo method integrating geodesics from the emitting source to the distant observers, and, on the other hand, camera techniques with backwards integration from the observer to the emission region. We present several test cases which our code successfully passed, including thin accretion disks around black holes and neutron stars hot spot emission.

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Context

The propagation of radiation in an arbitrarily curved spacetime is described by the covariant radiative transport equation

$$\frac{d}{d\lambda}\left(\frac{I_\nu}{\nu^3}\right) = \frac{j_\nu}{\nu^2} - \nu\alpha_\nu\left(\frac{I_\nu}{\nu^3}\right)$$

where I_ν is the specific intensity of the radiation field, j_ν and α_ν are the emission and absorption coefficients respectively, and $d/d\lambda$ is the convective derivative along the null geodesics of the spacetime. Each term in the equation is Lorentz-invariant. The equations of the geodesics

$$\frac{dx^\alpha}{d\lambda} = k^\alpha, \qquad \frac{dk^\alpha}{d\lambda} = -\Gamma^\alpha_{\mu\nu} k^\mu k^\nu \qquad \Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu})$$

where $g_{\mu\nu}$ is the spacetime metric and $\Gamma^\alpha_{\mu\nu}$ is the affine connection. In vacuum, where the right-hand side of the transport equation is zero, the transport simply reduces to the geodesic invariance of the ratio I_ν / ν^3. In this case, we only need to obtain the geodesics (ray tracing) and then we simply connect the emission between the initial and final points using the invariant I_ν / ν^3. **Skylight works in arbitrary spacetime geometries.**

Our main goal in developing Skylight is to use it in combination with the 3D general-relativistic force-free code Onion [1] to investigate the emission in systems with compact objects as pulsars, magnetars and binary systems, where the dynamics of the electromagnetic fields strongly determine the nature of the emission processes.

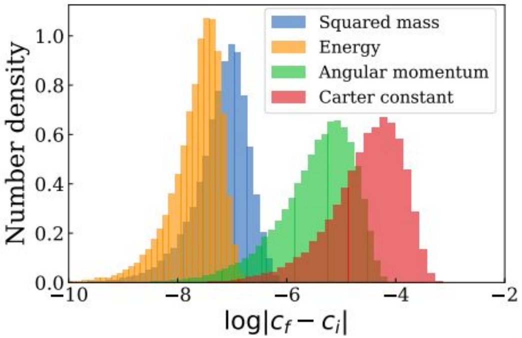
[1] Carrasco F. L., Reula O. A., 2017, Physical Review D, 96, 063006

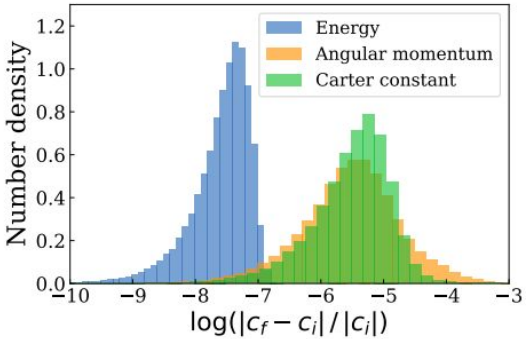
Ray tracing

We integrate the 8x8 system of first-order ODEs of the previous slide. This part is written in the relatively novel programming language **Julia**, a high-performance dynamically-typed language. Thus, we are able to achieve high performances while maintaining the ease of use. For the integration we use the package DifferentialEquations.jl [2] which provides a wide variety of built-in algorithms for the numerical solution of ODEs, much wider than traditional libraries. Apart from the standard algorithms, it includes many algorithms which are the result of recent research and are known to be more efficient than the traditional choices. We mostly use the method VCABM, an adaptive-order adaptive-time Adams-Moulton method, which is a good choice for high accuracy in very large systems.

Step size adaptivity is useful to accommodate to the needs depending on the local intensity of the gravitational field. The relative and absolute tolerances and the maximum step can be set as parameters of the solver method.

As a **verification**, we checked absolute and relative errors in the conservation of the four constants of motion in Kerr spacetime with spin a=0.99 for many different initial conditions. The tolerances of the solver are set to 10^{-8}. The results are shown below:





[2] Rackauckas C., Nie Q., 2017, Journal of Open Research Software, 5

Transport schemes

Emitter-to-observer

- Sample the local emissivity as a distribution of photon packets following

$$dN = \frac{dn}{w} = \frac{1}{w}\frac{j_\nu}{h\nu}\sqrt{-g}d^4x d\nu d\Omega$$

where w is a relative weight assigned to each packet and $d\Omega$ is the emission solid angle element. All quantities are referred to the local comoving frame, in which the emitting material is locally at rest.

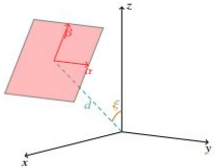
- Propagate the packets up to a large distance and collect them in bins over the celestial sphere
- Calculate the flux for each observer (each bin) as

$$F_\nu = \frac{1}{D^2 \Delta\Omega \Delta t \Delta\nu} \sum_i (h\nu)_i w_i$$

where $\Delta\Omega$, Δt and $\Delta\nu$ are the sizes of the bins in observer solid angle, time and photon frequency respectively

Observer-to-emitter

- For a given distant observer, prepare an initial data set of photons over an image plane with momenta perpendicular to the plane



- Trace the rays backwards in time to obtain their specific intensities from their values at the source

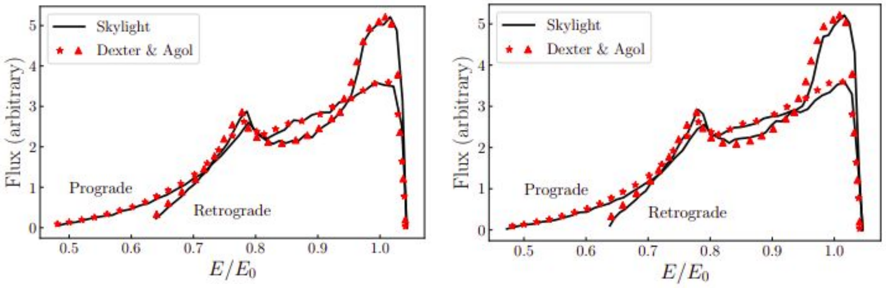
- Calculate the flux by integrating over the image plane as

$$F_\nu(t) = \frac{1}{D^2}\int_S I_\nu(\alpha, \beta, t) d\alpha d\beta$$

Astrophysical tests

Thin accretion disk around a Kerr black hole

The inner edge of the disk is at the ISCO and the outer radius is set to 15M. The black hole spin is a=0.5. The emissivity is monochromatic and decays as r^{-2}, and the local angular speed is that of the equatorial circular geodesics. Below we show the spectrum at inclination 30°, for prograde and retrograde disks in both the observer-to-emitter and emitter-to-observer schemes respectively, and compare with Dexter & Agol (2009).

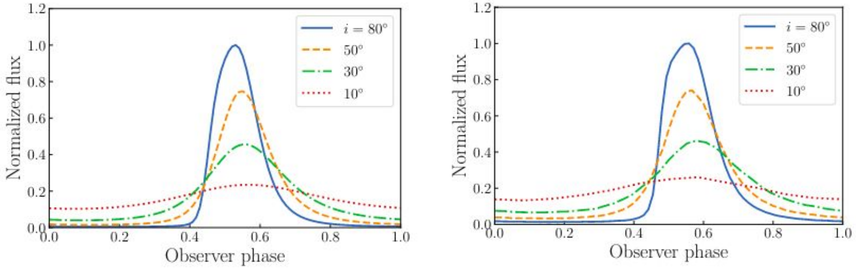


Neutron star hot spot emission

We consider the emission from circular hot spots over the surface of a rotating neutron star of 1.4 solar masses and radius 12km at a distance of 200 pc. Following the model of Bogdanov (2019), the emission is Planckian, corresponding to kT=0.35 keV, and it is isotropic We show the light curves in absolute units for various configurations in the observer-to-emitter scheme. The rest of the parameters for each case are shown in the Table.

Hot spot orbiting a Schwarzschild black hole

The center of the spot orbits at a radius of 6M on the equatorial plane with a Keplerian angular speed. The size of the spot is 0.25M. The emissivity is monochromatic and follows a Gaussian profile with the (Euclidean) distance to the center. Below I show the light curves for various inclinations obtained with the observer-to-emitter and the emitter-to-observer schemes. The agreement with Schnittmann & Bertschinger (2004) is very good, as is the mutual agreement between the schemes.



Parameter	Test SD1c	Test SD1d	Test SD1e
Colatitude of the spot center (°)	90	90	60
Angular radius of the spot (rad)	0.01	1	1
Colatitude of the observer (°)	90	90	30
Rotation frequency (Hz)	200	200	400