Galaxy rotation curve fitting using state-of-the-art machine learning tools

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Abstract*:* Nowadays machine learning is a tremendously powerful tool to solve a lot of different problems. In this work, we will use a specific machine learning tool known as *gradient descent* to fit the observed Galaxy's rotation curve. We will perform this fitting by assuming a theoretical velocity profile, arising from a composite model which includes baryons and a fermionic dark matter component. The last one explains the Galactic halo through a semi-analytical model of self-gravitating quantum fermions under the frame of general relativity. It has four free parameters including the particle mass which, in addition to the free parameters of the baryons, will be constrained by minimizing a loss function through the aforementioned gradient descent method.

Photo: https://es.m.wikipedia.org/wiki/Archivo:The_Andromeda_Galaxy.jpg

Introduction

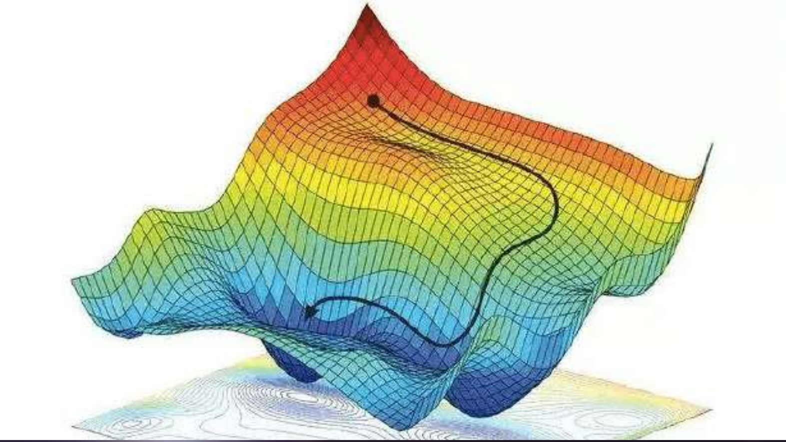
Exploring the parameter space of a given problem is a very important matter in every mathematical problem. In a physical context, the exploration of such a space gives us important information regarding the constraints imposed over the studied system. This was the motivation to develop methods to make those explorations. In this poster we will work on one particular method called *gradient descent* (GD), developed first by Louis Augustin Cauchy in 1847 [1].

Gradient descent

The gradient descent method is based on a progressive sequence of steps to minimize a function. Given a function *F* to minimize, it will implement the formula

$$\mathbf{p}^{\mathrm{new}} = \mathbf{p}^{\mathrm{old}} - \gamma \nabla_{\mathbf{p}} F(\mathbf{p}^{\mathrm{old}}) \quad (1)$$

where **p** stands for the independent variable of the function *F* and γ is a parameter called *learning rate* whose aim is to regulate the "length" of the steps. If this formula is implemented recursively, one eventually can go closer and closer to a minimum of *F*. An illustration of the procedure followed by the gradient descent method is shown in the figure to the right. In that image can be seen how the path followed by the method finds the deepest point of the function going in the direction opposite to the function's gradient.



In black the path defined by the sequence of steps given by the formula (1) and in colors the surface defined by a generic function *F*.

Photo: https://easyai.tech/en/ai-definition/gradient-descent/

Application of the method: The Galaxy rotation curve

Applied to our interests, the function *F* of the previous page will be a function that quantizes how good are the predictions made by the model in contrast to the observations. In machine learning, these kind of functions are called *loss functions*. Specifically, we will use a Mean Squared Error (MSE) as a loss function, which is defined as:

$$\mathrm{Loss}(\mathbf{p}) = \frac{1}{C}\sum_{i=1}^{N}\frac{(V_i(\mathbf{p}) - v_i)^2}{N}$$

where *C* is a normalization constant, *N* is the number of observations, **p** are the physical free parameters that characterize the model, V are the predicted circular velocities and *v* are the observed ones. The idea is to fit the free parameters of the model to the rotation curve given in Sofue 2017 [2]. But before we proceed to explore new frontiers, it is convenient to test the method against the results of previous works.

Testing the method

To test the method we will adopt the Galaxy's potential model used in Argüelles et al. 2018 (henceforth PDU18) [3]. In that work is assumed that the potential of the Galaxy is composed by two components, a baryonic component and a fermionic dark matter (DM) one. They adopt as baryonic component one composed by two bulges (inner and main) and one flat disk. The density of the bulges are modeled as exponential spheroids and the disk surface mass density is modeled as an exponential disk. The formulas of such models are given in the table to the right.

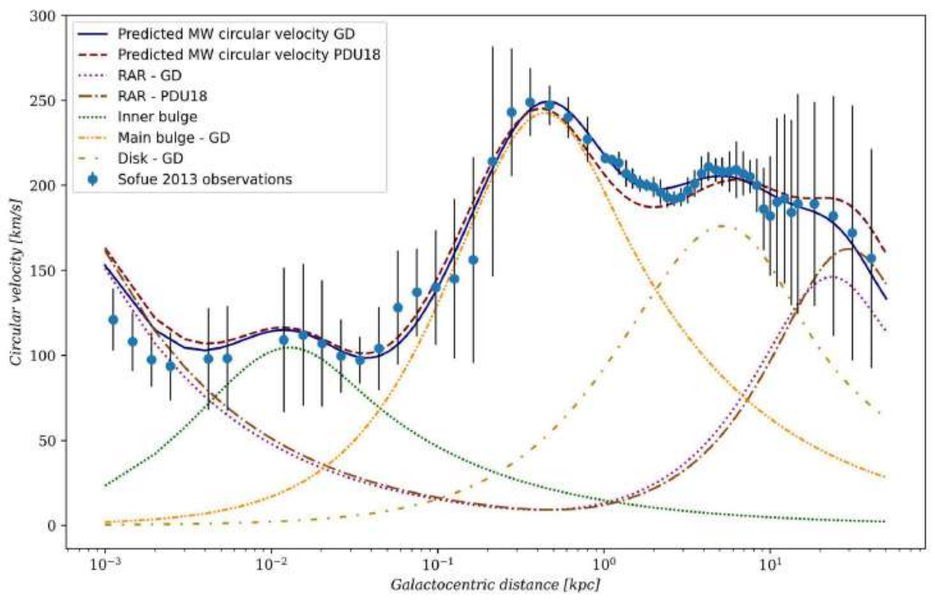
Bulge	$\rho(r) = \rho_c e^{-r/a_b}$
Disk	$\Sigma(R) = \Sigma_0 e^{-R/a_d}$

Dark halo model

To model the dark component of the Galaxy it is used a semi-analytical model based on a self-gravitating system of quantum fermions with particle scape effects under the frame of general relativity, usually known as *Ruffini - Argüelles - Rueda (RAR) model with cut-off* [3]. This model has four free parameters, the mass of the DM particle, and three regarding the chemical potential θ_0, the cut-off energy W_0 and the temperature of the system β_0 in its center ($r = 0$). To get the mass profile of such a system it has to be solved the Tolman–Oppenheimer–Volkoff equations in addition with an equation of state and the Tolman [4] and Klein [5] conditions. See [3] for a detailed explanation. The solution of the RAR system of equations gives a halo with three different regimes: a quantum dense core which mimics the central supermassive black hole behaviour, a sharply decreasing density distribution followed by an extended plateau and a Boltzmannian density tail.

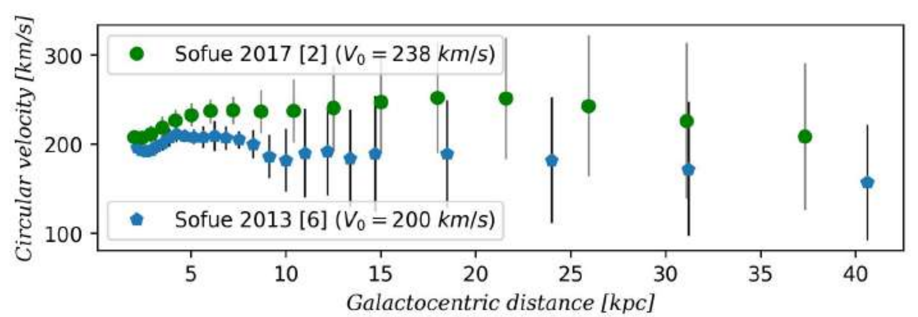
Comparison with the rotation curve got in Argüelles et al. 2018

If we apply the gradient descent method to fit the free parameters of the main bulge, disk and θ_0 and β_0, we will get a rotation curve (RC) as shown to the right. It can be seen that the fit is more accurate than in the case of [3]. The dataset of circular velocity points used is the one given in Sofue 2013 [6] as used in [3].



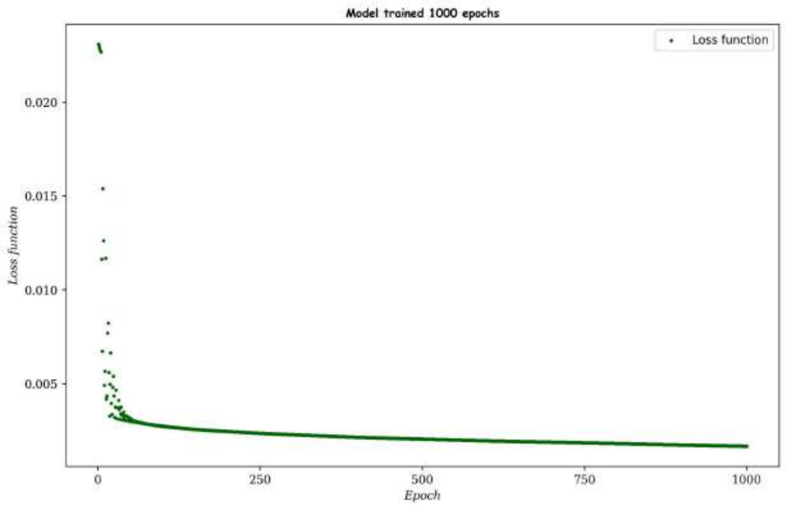
Fitting the Galactic potential using more accurate observations

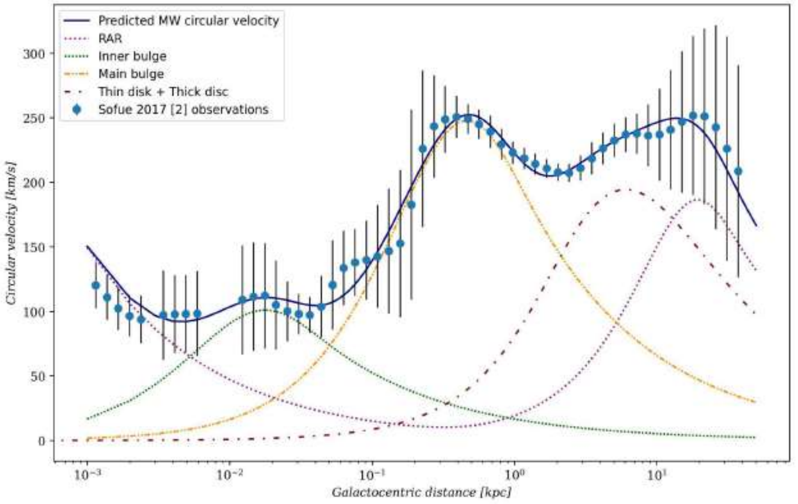
Studies of the Milky Way parameters R_0 and V_0 [1] in the last few years have determined these values to be approximately 8 kpc and 238 km/s respectively [2]. In [6] it was used the values 8 kpc and 200 km/s, leading to a wrong observed rotation curve in the outskirts of the Galaxy as seen to the right.



Considering these new values, it was used the RC given in [2] to fit the Galactic potential assuming it is composed of two exponential spheroid bulges (inner and main), two Miyamoto-Nagai disks (thin and thick) and a RAR halo.

The result of this fitting and the corresponding loss function are shown to the right.





[1] *Distance from the Sun to the galactic center and circular velocity of the LSR, respectively.*

Changes in parameters

To let the code fit the parameters, it was given a *seed of good parameters* to help the GD method find the minimum of the loss function in an easier way. Each bulge provided two parameters, each disk three more, and the RAR model provided four free parameters. The changes in the parameters after 1000 epochs or steps are listed in the table below.

ρ_c^{inner}	a_b^{inner}	ρ_c^{main}	a_b^{main}	a_{MN}^{thin}	b_{MN}^{thin}	M_{MN}^{thin}
$-51.23\,\%$	$+38.34\,\%$	$-18.72\,\%$	$+14.62\,\%$	$+25.25\,\%$	$+25.25\,\%$	$+17.31\,\%$
a_{MN}^{thick}	b_{MN}^{thick}	M_{MN}^{thick}	m	θ_0	W_0	β_0
$-3.82\,\%$	$-3.82\,\%$	$+7.15\,\%$	$-14.45\,\%$	$+0.09\,\%$	$+0.61\,\%$	$-23.74\,\%$

Conclusions

To conclude, we can remark the power and the accuracy of the method since it has given a best-fit of the RC of the Galaxy based on well accepted observations assuming a gravitational potential constituted of **fourteen** free parameters, four of them belonging to a semi-analytical DM halo model. Also, it is important to mention the speed of the method, since it has taken about one and half an hour in conclude the 1000 epochs. It is due since the GD method makes the steps go in the direction opposite to the gradient, resulting in a direct method to find the minimum of the loss function. If it is checked the plot of the loss function it can be seen that this function has a steep decreasing near 100 epochs, suggesting that the convergence of the method may be faster than thought. Regarding the physics of the problem, it is important to notice that the RC is well fitted in galactocentric distances greater than ~2 pc. Inside this distance, the effects of the quantum core are stronger and other tracers need to be used to fully constrain the free parameters of the RAR model [3].

References

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